COMPARISON OF DEFLECTION FOR PLATE WITH CENTRAL CIRCULAR HOLE BY EIGHT NODED ISOPERIMETRIC PLATE ELEMENT WITH OTHER METHODS

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ABSTRACT *: In recent studies (1) the author has presented the use of Hinton's eight noded isoparametric plate element for analysis of perforated plate. Comparison of deflection for simply supported square plate with central circular hole loaded with uniform transverse load found out by using the eight noded isoparametric plate element with the existing results is the main aim of this studies. Problems of plate with hole arises in different engineering study. The most apparent feature of the isoparametric elements are sides that may be curved and their special coordinates (,) system. They are useful in modelling structures with curved edges and in grading a mesh from coarse to fine. Hence, Hinton's eight noded isoparametric plate element is thought to be most suitable for the analysis. The results are presented in the form of graph and table.*

KEYWORDS : Hinton's Element, Isoparametric, Special Coordinate, Mindlin Plate Theory.

INTRODUCTION :

The plate with holes of different shapes and sizes are used in different engineering structures. Researchers are using different method to arrive at the solution and present the data in various forms. In the present study the eight noded isoparametric plate element developed by Hinton which was based on Mindlin plate theory is used to analyse the plate with central circular hole and the results of deflection are compared with the results presented by Seide and Chang (2), Chaudhari and Seide (3) and El-Hashimy's (4) theory.

Isoparametric elements were publicly introduced in 1966. They make it possible to have nonrectangular quadrilateral elements. Their most apparent features are sides that may be curved and their special coordinates (ξ,η) system. Isoparametric elements are useful in modelling structures with curved edges and in grading a mesh from coarse to fine. Isoparametric elements are versatile.

In 1973 Hinton (5) developed an isoparametric plate element in cartesian coordinates based on Mindlin plate theory. It is convenient to use Hinton's element for analysis of plate with hole. This eight noded Isoparametric plate element, based on thick plate theory, in which normal remains straight but does not remain normal to the middle plane, enhanced the applicability of finite element method for modelling perforated plates with various boundary conditions. The element has 3 d.o.f. per node, i.e. transverse displacement and rotations of the normal about X and Y axes, resulting in 24 x 24 stiftness matrix. The bending, twisting and shear energies are evaluated by 2x2 integration scheme.

With this background, the square plate with simply supported condition and uniformly distributed transverse load and weakened by central circular hole is considered for analysis. The results of the deflections are compared with the available results of finite element analysis by Seide and Chang (2), Chaudhari and Seide (3) and El-Hashimy's (4) analysis and presented in the form of graph and table. It is concluded that the present analysis converges to the correct solution of the plate with hole.

PLATE DESCRIPTION :

A homogeneous isotropic plate with all edges simply supported is consider for analysis. Only cartesian coordinates for geometric modelling of the plate with central circular hole have been used. The symmetry of the plate about X and Y axes allows to analyse only quarter of the plate. The position of hole selected is such that the symmetry condition can not be violated for perforated plate. The Symmetry conditions are, (i) rotation of normal about X-axis $\theta x = 0$, along X - axis and (ii) rotation of normal about Y - axis $\theta y = 0$, along Y - axis. At simple support the boundary condition is displacement along Z-axis, $w = 0$ and free edge condition at the boundary of the hole.

The constant numerical data adopted for analysis of plate with central circular hole is

Size of plate, 2000mm x 2000mm

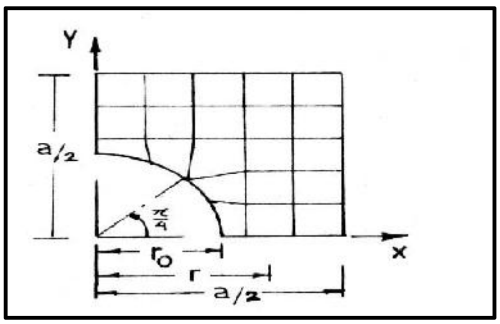
Thickness = 10mm
Youngs modulus,
E=2.1 x 10^5 N/mm^2

Uniformly distributed load,
q_z= 10 KPa

and poissons ratio,
μ = 0.3

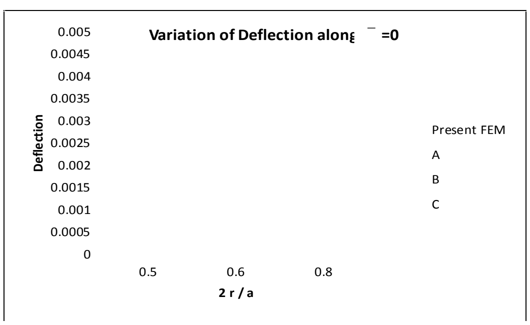
A typical finite element model for a simply supported square plate with central circular hole is as shown in Fig. 1.

Analysis is done through simulation for eight noded isoparametric plate element in cartesian coordinate. Joint displacement code number approach is used for assembly of element stiffness matrices into structure stiffness matrix. This is found to be quite convenient as zero code numbers are assigned to the restrained displacements. This equations are formed corresponding to the unrestrained displacement only. The input consist of material properties, geometrical data for plate and hole, loading etc. The coordinates of the nodes and code numbers are generated by simple technique so as to reduce the data. The structure stiffness matrix is assembled in half band form. The output consists of the nodal displacements and stress resultants at Gauses points. The results of deflections are compared with the available results in graphical form in Fig. 2 and tabular form in Table 1.



All Edges Simply Supported	
Plate size a	= 2000mm
Radius of hole r_o	= 500mm
Thickness t	= 10 mm
u.d.l. q_z	= 10 Kpa

Fig. 1. : Finite Element Model of S.S. SQ. Plate with Central Circular Hole



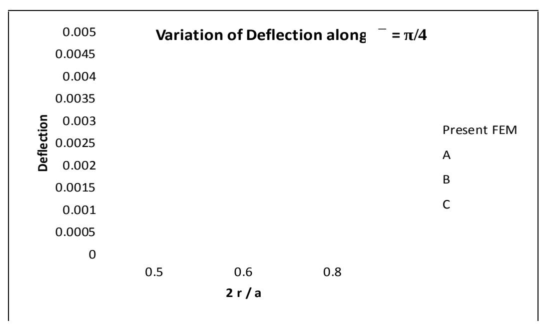


Fig. 2. : Graphical Representation of Deflection Coefficient for S.S. Sq. Plate with Central Circular Hole.

Sr. No.	2r/a	θ = 0				θ = π/4			
		Present F.E.M	A (3)	B (4)	C (2)	Present F.E.M	A (3)	B (4)	C (2)
1	0.5	0.00325	0.00471	0.00458	0.00207	0.00333	0.00484	0.00470	0.00213
2	0.6	0.00263	0.00382	0.00371	0.00167	0.00281	0.00405	0.00394	0.00182
3	0.8	0.00134	0.00198	0.00181	0.00858	0.00175	0.00258	0.00249	0.00117

Table 1. : Deflection Coefficient in Tabular Form for S.S. Sq. Plate with Central Circular Hole.

COMMENTS ON RESULTS :

From the study of results obtained from the present analysis and comparison from the Table 1 and Fig.2 it is observed that the deflection obtained from the present analysis is greater than those obtained from the assumed linear displacement finite element analysis of Seide and Chang (2) and smaller than the deflection obtained from the finite element analysis using quadratic displacement triangular element with straight sides of Chaudhari and Seide (3) and using El-Hashimy's (4) theory. It can be seen that the transverse displacements of the linear element due to Seide and Change (2) are yet to converge to the correct solution, because this element is too stiff in bending. Also, Chaudhari and Seide (3) obtained the results using an assumed quadratic displacement element with straight sides. This element behave like a subparametric element and when this element version is used to model the curved boundary of the hole, there is a change of domain which results in boundary layer effect. It should be rembered that El-Hashimy's solutions have been obtained by satisfying the boundary conditions only at three points of the plate boundary and boundary of the hole. Therefore a traction free edge condition is not satisfied in general. El-Hashimy has neglected shear.

CONCLUSION :

The results for deflection, along $\theta=0$ and $\theta=\pi/4$, obtained in the present study are compared with the results obtained for deflection with same characteristics of the plate with central circular hole found out by other authors with different methods. The most apparent feature of the isoparametric elements are sides that may be curved and are useful in modelling structures with curved edges and in grading a mesh from coarse to fine. Hence, the result from the present analysis are more accurate even near the vicinity of the hole. The difference between the deflection between different methods goes on diminishing away from the boundary of the hole. The study can be further extended to find out the maximum deflection, bending moments and stress concentration factors for the plates with more than one hole and different size, shape and combination of holes.

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