IMAGE RESTORATION USING CURVELET TRANSFORM

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ABSTRACT: *Wavelet transform has played an important role in image processing tasks such as compression and restoration. But, it fails to represent effectively the images, which have edges and treated them as smooth functions with discontinuities along curves. In this implementations we get exact reconstruction, stability against perturbations, and low computational complexity. The curvelet transform has been developed in which frame elements are indexed by scale, location, and orientation parameters. The elements obey a special scaling law, where the length of the support of frame elements is approximately equal to square of the width of the support. Curvelet transform is designed to represent edges and other singularities along the curves much more efficient than the traditionally transform. Here in this paper we are mainly focusing on noise not on blur part of the image.*

Keywords— Ridgelet transform , curvelet transform , radon transform , rectopolar .

I: INTRODUCTION

In the past, many of the wavelet methods for restoration in signals and images are introduced. Firstly very simple ideas like thresholding of the orthogonal wavelet coefficients of the noisy data, followed by reconstruction is introduced after that some improvements introduced in perceptual quality could be obtained by translation invariant methods based on thresholding of an undecimated wavelet transform.

In dimensions two or higher, wavelets can efficiently represent only small range of the full diversity of interesting behavior. In effect, wavelets are well adapted for point like phenomena, where in dimensions greater than one, interesting phenomena like ridgelet and curvelet transform can be organized for lines, hyper planes, and other non point structures, for which wavelets are poorly adapted.

Imagine an object supports in unit square [0,1]x[1,0] having discontinuity across a smooth curve. How fast can we approximate it using certain system of functions? Let f_n^F be the best partial reconstruction obtained by selecting the *n* largest in the Fourier series; then the squared error of such an *n* – term expansion satisfied

$$|f - f_n^F||_{L^2}^2 \sim n^{-0.5} \quad , \quad n \to \infty$$

Wavelet have an improved rate of approximation. The approximant f_n^W built from the best *n* nonzero wavelet term satisfies

$$|f - f_n^W||_{L^2}^2 \sim n^{-1} \quad , \quad n \to \infty$$

With curvelet transform , the approximant f_n^C built from the best *n* nonzero wavelet terms satisfies

$$|f - f_n^C||_{L^2}^2 \sim n^{-2} \quad , \quad n \to \infty$$

The curvelet system gives a good result then wavelet system.

II: THIS PAPER

According to theory discrete ridgelet transforms and discrete curvelet transforms provide near-ideal sparsity of representation of both smooth objects and of objects with edges. In this paper we provide an initial test of these ideas in a digital image processing setting, where images are available on an N X N grid. We first review some basic ideas about ridgelet and curvelet representations in the continuum. We next use these to develop a series of digital ridgelet and digital curvelet transforms taking digital input data on Cartesian grid. Next we consider a model restoration problem where we embed some standard images in Gaussian noise and then apply thresholding in the digital curvelet transform domain. Finally we discuss interpretations and possibilities for future work.

III: CONTINUOUS RIDGELET TRANSFORM

The continuous ridgelet transform (CRT) in $\boldsymbol{R}^2$ can be defined as follows [4]. Pick a smooth univariate function : $\boldsymbol{R} \rightarrow \boldsymbol{R}$ with sufficient decay and vanishing mean, each b € $\boldsymbol{R}$ and each θ € [0,2Π) , define the bivariate function $\varphi_{a,b,\theta}$: $\boldsymbol{R} \rightarrow \boldsymbol{R}$. [19]

$$\varphi_{a,b,\theta}(x) = a^{-0.5}\, \varphi\left((\cos(\theta)\, x_1 + \sin(\theta)\, x_2) - \frac{b}{a} \right) \quad (1)$$

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This function is constant along "ridges" $\cos(\theta)x_1 + \sin(\theta)x_2 = const$. Perpendicular to these ridges it is known as a wavelet; so the name ridgelet. If an integrable bivariate function $f(x)$ is given, its ridgelet coefficients are defined as

$$R_f(a,b,\theta) = \int \varphi_{a,b,\theta}(x) f(x)\, dx \qquad (2)$$

The reconstruction formula is

$$f(x) = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\, \varphi_{a,b,\theta}(x)\, \frac{da}{a^3}\, db\, \frac{d\theta}{4\pi} \qquad (4)$$

valid for functions that square integrable, which shows that function can be written as a superposition of 'ridge' functions. This representation is stable as it has a Parseval relation:

$$\int |f(x)|^2 dx = \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\, \varphi_{a,b,\theta}(x)\, \frac{da}{a^3}\, db\, \frac{d\theta}{4\pi} \qquad (5)$$

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IV: RADON TRANSFORM

A basic tool for calculating ridgelet coefficients is to view ridgelet analysis. In 2D , point and line are related via radon transform and hence wavelets and ridglet transform are linked via radon transform. The radon transform is defined as

$$Rf(\theta,t) = \int f(x_1,x_2)\, \delta(x_1\cos\theta, x_2\sin\theta - t)\, dx_1 dx_2 \qquad (6)$$

Where δ is daric function. The ridgelet coefficient $R_f(a,b,\theta)$ of an object f are given by applying 1D wavelet transform to the pieces of the Radon transform as :

$$R_f(a,b,\theta) = \int Rf(\theta,t)\, a^{-0.5}\varphi\left(\frac{t-b}{a}\right) dt \qquad (7)$$

V: RIDGELET PYRAMIDS

Let Q denote a dyadic square $Q = [k_1/2^s, (k_1+1)/2^s) \times [k_2/2^s, (k_2+1)/2^s)$ and let Q be the collection of all such dyadic squares. We write Qs for the collection of all dyadic squares of scale s. Associated to the squares $Q \in Qs$ we construct a partition of energy as follows. With w a nice smooth window obeying $\sum k_1, k_2\, w^2(x_1 - k_1, x_2 - k_2) = 1$, we dilate and transport w to all squares Q at scale s, producing the collection of windows (w_Q) such that the w_Q's, $Q \in Qs$, make up the partition of unity. We also let T_Q denote the transport operator acting on functions [19]

$$(T_Q g)(x_1,x_2) = 2^s g(2^s x_1 - k_1, 2^s x_2 - k_2) \qquad (9)$$

$$fw_Q = \int \langle f, w_Q T_Q \varphi_{a,b,\theta} \rangle\, T_Q\varphi_{a,b,\theta}\, \frac{da}{a^3}\, db\, \frac{d\theta}{4\pi} \qquad (10)$$

Summing the above quantity across squares at a given scale gives

$$f = \sum f w_Q^2 = \int \langle f, w_Q T_Q \varphi_{a,b,\theta} \rangle\, T_Q\varphi_{a,b,\theta}\, \frac{da}{a^3}\, db\, \frac{d\theta}{4\pi} \qquad (11)$$

This shows that we can represent any function as a superposition of elements of the form $w_Q T_Q \varphi_{a,b,\theta}$ that is, of ridgelet elements localized near the squares Q.For the function $T_Q \varphi_{a,b,\theta}$ is the ridgelet $\varphi_{a,b,\theta}$ such that parameters obeys

$$a_Q = 2^{-s}a, \theta_Q = \theta,\ b_Q = b + k_1 2^{-s}\cos\theta + k_2 2^{-s}\sin\theta \qquad (12)$$

$w_Q T_Q \varphi_{a,b,\theta}$ is is a windowed ridgelet, supported near the square Q, hence the name local ridgelet transform. If the scale s is varied , we get multiscale ridgelet dictionary as follows , which is a whole pyramid of local ridgelet at various lengths and locations. [19]

VI: APPROXIMATE DIGITAL RIDGELET TRANSFORM

Basic strategy for calculating the continuous ridgelet transform is first to compute the Radon transform Rf(t; µ) and second, to apply a one-dimensional wavelet transform to the slices Rf(¢; µ). In this section we develop a digital procedure which is inspired by this viewpoint, and is realizable on n by n numerical arrays,

Fourier Strategy for Digital Radon Transform

A fundamental fact about the Radon transform is the projection formula [11]:

$$f(\lambda\cos\theta, \lambda\sin\theta) = \int Rf(t,\theta)\, e^{-i\lambda t}\, dt \qquad (13)$$

This says that the Radon transform can be obtaine by applying the 1D inverse Fourier transform to the 2D Fourier transform restricted to radial lines going through the origin.

This is a widely used approach, in the literature of medical imaging and synthetic aperture radar imaging, for which the key approximation errors and artifacts have been widely discussed. In outline, one simply does the following,

for gridded data (f(i1; i2)), 0 < i1; i2 < n-1.

1. *2D-FFT.* Compute the two-dimensional FFT on the giving array.
2. *Cartesian to Polar Conversion.* Using an interpolation scheme, substitute the sampled values of the Fourier transform obtained on the square lattice with sampled values of f' on a polar lattice: that is, on a lattice where the points fall on lines going through the origin.
3. *1D-IFFT.* Compute the one-dimensional IFFT on each line.

The use of this strategy in connection with ridgelet transforms has been discussed in the articles.

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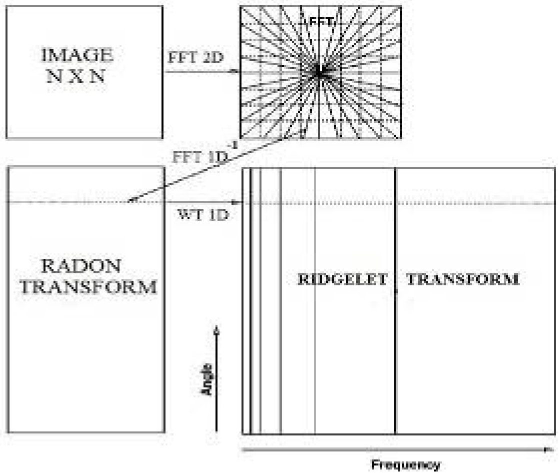

Fig.1 Ridgelet transform flow graph

VII: CONSTRUCTION OF CURVELETS

Curvelet transform is a new multiscale transform designed to represent edges and other singularities along the curves much more efficient than the traditionally transform. The curvelet transform like wavelet transform is multiscale transform with frame elements indexed by scale and location parameter. Unlike the wavelet transform, it has directional parameter , and curvelet pyramid contains elements with a very high degree of directional specificity. In addition , the curvelet transform is based on a certain anisotropic scalling principle ,which is quite different from the isotropic scalling of wavelets. The elements obey a special scalling are linked by the relation *width = length*^{2} .

At level j the support of the wavelet basis has a size 2^{-j}. The curve scalling relation suggests that we can group $C_2^{-j/2}$ wavelet basis function into one basis function with linear structure so that *width = length*^{2}.

Thus , curvelet basis function can be viewed as local grouping of wavelet basis function so that they capture the smooth discontinuity curve more efficiently.

Curvelets give optimally sparse representation of otherwise smooth objects. The curvelet transform [1,2] is derived by combining various notions:

- *Ridgelets:* Method of analysis suitable for objects with discontinuities across straight lines.
- *Multiscale Ridgelets:* Pyramid of windowed ridgelets, renormalized and transported to a wide range of scales and locations.
- *Bandpass Filtering (Subband filtering):* Method of separating an object out into the series of disjoint scales. And decompose f into subbands using the standard filterbank concept. Then one specific monoscale dictionary $_{MS}$ to analyze one specific subband. Defined coronae of frequencies $|\zeta| \epsilon\ [2^{2s}, 2^{2s+2}]$, and subband filters $_{Ds}$ extracting components of f in the indicated subbands; a filter $_{Ps}$ deals with frequencies $|\xi| \leq 1$. The filters decompose the energy exactly into the subbands:

$$\|f\|_2^2 = \|P_0 f\|_2^2 + \sum_s \|D_s f\|_2^2 \qquad (14)$$

The construction of such operators is standard ; the coronization oriented around powers 2^{2s} is nonstandard and essential here. Explicitly, a sequence of filters $\emptyset_0$ and $\varphi_{2s} = 2^{4s}\varphi(2^{2s}), s = 1,2,3, ...$ with the following properties $\emptyset_0$ is a lowpass filter concentrated near frequencies $|\xi| < 1$; φ_{2s} is bandpass, concentrated near $|\zeta| \epsilon\ [2^{2s}, 2^{2s+2}]$ and there is

$$|\phi_0(\zeta)|^2 + \sum_{s2} |\varphi(2^{-2s}\zeta)^2| = 1 \qquad (15)$$

Hence, D_s is simply the convolution operator $Ds\,f = \psi_{2s} * f$.

The curvelet transform can be described as a combination of reversible transformations as follows:

- 2D Wavelet transform
- Smooth partitioning
- Ridgelet transform
  - Radon transform
  - 1D Wavelet transform

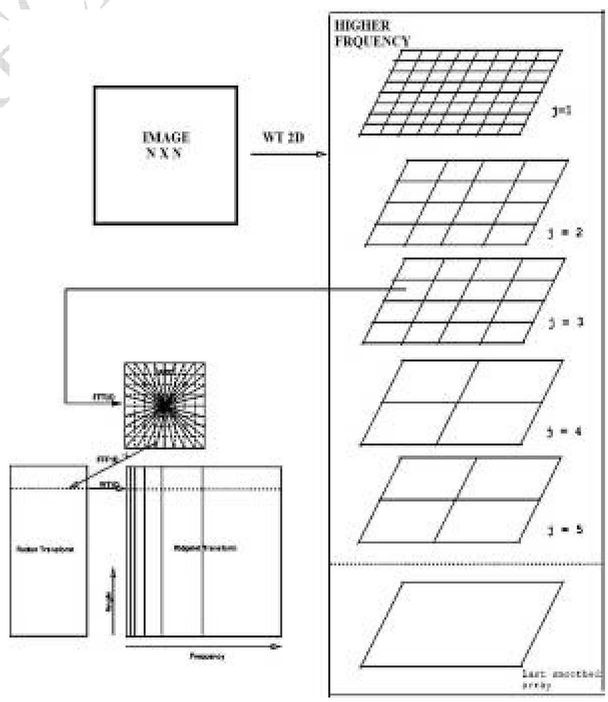

Fig 2 .flow chart of curvelet transform

VIII: ANALYSIS OF THE CURVELET TRANSFORM

The curvelet decomposition can be stated in the following steps:

1. Subband Decomposition: Where the object f is filtered into subbands

$$f \rightarrow (P_0 f, \Delta_1 f, \Delta_2 f \ldots.) \qquad (16)$$

2. Smooth Partitioning: Each subband is smoothly windowed into "squares" of an appropriate scale:

$$\Delta_1 f \rightarrow (W_Q \Delta_s f\ )_{Q \in Q_s} \ldots \qquad (17)$$

Where Q denote dyadic square $Q=[k/2^s,(k+1)/2^s)] \times [k/2^s,(k+1)/2^s)]$ and let Q be the collection of all such dyadic squares. The notation Q_s will correspond to all dyadic squares of scale s. Let W_Q be a window centered near Q, obtained after dilation and translation of a single w , such that the W_Q^2 , $Q \in Q_s$, make up a partition of unity.

3. Renormalization: Each resulting square is renormalized to unit scale

$$g_Q = (T_Q)^{-1}\left(W_Q\,\Delta_s f\right),\ \ Q \in Q_s \qquad (18)$$

The multiscale ridgelet system renormalizes and transports the ridgelet basis method, so that one has a system of elements at all lengths and all finer widths.

4. Ridgelet Analysis: Where the orthonormal ridgelet transform is applied to each square. This is a system of basis elements P_λ on making an orthobasis.

$$a_U = \langle g_Q, P_\lambda \rangle\ , u = (Q, \lambda) \qquad (19)$$

Two dyadic subbands $[2^{2s}, 2^{2s+1}]$ and $[2^{2s+1}, 2^{2s+2}]$ are merged before applying the Ridgelet transform.

IX: SYNTHESIS OF THE CURVELET TRANSFORM

The procedural definition of the reconstruction algorithm

1. Ridgelet Synthesis: Each 'square' is reconstructed from the orthonormal ridgelet system

$$g_Q = \sum_\lambda a(\lambda, Q)\rho_\lambda \qquad (20)$$

2. Renormalization: Each 'square' resulting in the previous stage is renormalized to its own proper square

$$g_Q = (T_Q) g_Q\ ,\ \ Q \in Q_s \qquad (21)$$

3. Smooth Integration: Reverse the windowing dissection to each of the windows reconstructed in the previous stage of the algorithm.

$$\Delta_s f = \sum_{Q \in Q_s} W_Q h_Q \qquad (22)$$

4. Subband Recomposition: Undo the bank of subband filters, using the reproducing formula:

$$f = P_0(P_0 f) + \sum_{s>0}^{J} \Delta_s(\Delta_s f). \qquad (23)$$

X: DISCRETE CURVELET ALGORITHM

The 'trous' subband filtering algorithem to the digital Curvelet transform. The algorithm decomposes the N X N image as a superposition of the form :

$$f(x,y) = c_J(x,y) + \sum_{j=1}^{J} w_j(x,y) \qquad (24)$$

Where C_J is the smooth version of the original image I and W_j represents the details I at scale 2^{-j} . The algorithm output J+1 subband arrays of size N X N. The indexing is such that j=1 corresponding to the finest scale (high frequency)

1. Apply the à trous algorithm with J scales
2. set $B_1 = B_{min}$
3. for $J = 1,2,\ldots.$ J Do

(a) partition the subband W_j with a block size B_j and apply the digital ridgelet transform to each block;

(b) if j modulo 2 = 1 then $B_{j+1} = 2B_j$

(c) else $B_{j+1} = B_j$

This implementation of the curvelet transform is also redundant. The redundancy factor is equal to 16J +1 whenever J scales are employed. Finally, the method enjoys exact reconstruction and stability, because this invertibility holds for each element of processing chain.

The sidelength of localizing the windows is doubled at every other dyadic subband, hence maintaining the fundamental property of the curvelet transform which says that the elements of length about $2^{-j/2}$ serve for the analysis and synthesis of the subband .Also note that the coarse description of the image C_J is not processed. Finally, Fig. 2 gives an overview of the organization of the algorithm.

XI: SIMULATION RESULTS

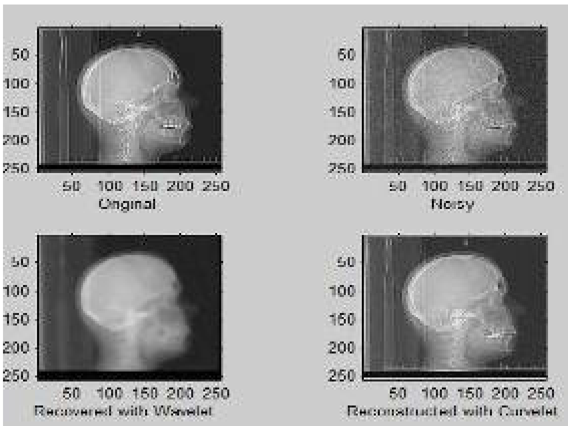

Fig 3 Simulation result for medical image

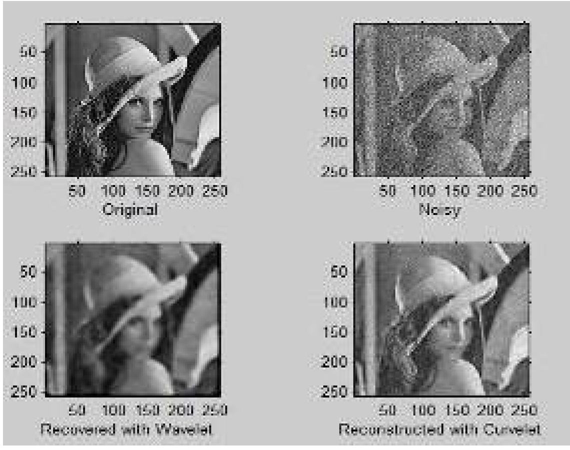

Fig 4 Simulation result for Lena image

Table of PSNR values after the restoration of noisy image [IMAGE + GAUSSIAN WHITE NOISE (SIGMA)] for the different value of sigma.

IMAGES	SIGMA	WAVELET METHOD (PSNR IN DB)	CURVELET METHOD (PSNR IN DB)
CAMERAMAN	10.59	21.3684	32.4320
	19.90	21.1492	29.1292
	42.36	20.3186	25.2319
LEENA	10.59	22.2322	32.5025
	19.90	21.9450	29.1463
	42.36	20.9441	25.2416
MEDICAL	10.59	18.7137	33.2129
	19.90	18.7179	29.7721
	42.36	18.4476	25.8676

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XII: CONCLUSION

From the above experiment & result analysis it is very clear that the curvelet method is best for the image restoration. Ridgelet transform methods which is very effective in representing object with singularities along line where the wavelet fails. Curvelet transform coefficient technique is conceptually simpler, faster and far less redundant than the existing technique in case of Gaussian noise irrespective of type of images

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