QUANTUM INFORMATION THEORY
EVOLUTIONARY PROBABILITY THEORY

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Abstract
Evolutionary Probability Theory is showed evolution of probability. Evolutionary Probability is based on Quaternion Algebra . Every experiment of probability provides to change our system. Known probability is a mathematical entity and scalar entity. But evolutionary probability will be a mathematical entity and vector entity that has both magnitude and direction. We start our discussion of evolutionary probability vectors by presenting a couple of different representations of vectors, magnitude and direction. We can apply on its for Quantum Information Theory.

1 Introduction

Graphically, Evolutionary Probability is a quaternion and vector is represented by an arrow on four dimensional space. The magnitude of the probability is represented by the length of the arrow and the direction of the probability is represented by which way the arrow is pointing. P probability variable is represented by a letter with an arrow over it as in the vector $\vec{P}$ .Once we define a vector, we often need to write about the magnitude of that vector just how big it is as opposed to both how big and which way it is. The magnitude of the probability P can be written two ways, either $\left|\vec{P}\right|$ or $|P|$. The first way makes it more obvious that we are dealing with the magnitude of a probability rather than some ordinary variable. The second method is used when it is already clear from the context that we are dealing with the magnitude of a probability [1,2,3,4,5,6,7,8,9,10,11,12,13,14].

2 Notations

2.1 Notations for Evolutionary Probability Theory

Given as set Ω set and subsets A, B, Ω then the following notation is used:

1- Intersection:
$A \cap B = \{v \in \Omega : v \in A and v \in B\}$

2- Union:
$A \cup B = \{v \in \Omega : v \in A or, or both v \in B\}$

3- Minus:
$A \setminus B = \{v \in \Omega : v \in A and v \notin B\}$

4- Complement:
$A^{c} = \{v \in \Omega : v \notin A\}$

5- Empty Set:
$\varnothing$ set without any element.

2.2 Components of Probability Space

Any probability space (Ω, F, P) consists of three components.

1- The elementary events or states w which are collected in a non-empty Ω set .

2- Any σ algebra F , which is the system of observable subsets or events $A \subseteq \Omega$ .The interpretation is that one can usually not decide whether a system is in the particular state $w \in \Omega$,but one can decide whether $w \in A$ or $w \notin A$.

3- Any measure P, which gives a probability to all $A \in F$.This probability is a number $P(A) \in [0,1]$ that describes how likely it is that the event A occurs. We define the σ algebras F, here we do not need any measure.

2.3 σ Algebras

Let be Ω a non-empty set. A system F of subsets $A \subseteq \Omega$ is called σ algebra on Ω if

1- $\emptyset, \Omega, F$

2- $A \in F$ implies that $A^c : \Omega \setminus A \in F$

3- $A_1, A_2, \ldots \in F$ implies that $\bigcup_{i=1}^{\infty} A_i \in F$

The pair (Ω, F), where F is a σ algebra on , is called measurable space. The elements $A \in F$ are called events. An event A occurs if $w \in A$ and it does not occur if $w \notin A$.

4- $A, B \in F$ implies that $A \cup B \in F$, then F is called an algebra. Every σ algebra is an algebra. Sometimes, the terms σ field and field are used instead of σ algebra and algebra.

3. Space Definition

3.1 Probability space

Let (Ω, F), be a measurable space.

1- A map $P : F \to [0,1]$ is called probability measure if $P(\Omega) = 1$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \emptyset$ for $i \neq j$ one has $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

The triplet (Ω, F, P) is called probability space.

2- Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\emptyset) = 0$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \emptyset$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3- Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,\ldots$, such that

a- $\Omega_k \in F$ for all $k = 1,2,\ldots$

b- $\Omega_i \cap \Omega_j = \emptyset$ for $i \neq j$

c- $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d- $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

3.2 Components of Evolutionary probability space

Any evolutionary probability space (Ω, F, P) consists of three components .But probability will have G theoretical and M_1, M_2, M_3 experimental probability components $P(G, M_1, M_2, M_3)$ on evolutionary space. Because of an evolutionary probability space $(\Omega, F, G, M_1, M_2, M_3)$ consists of six components.

1- The elementary events or states w which are collected in a non-empty Ω set .

2- Any σ algebra F, which is the system of observable subsets or events $A \subseteq \Omega$.The interpretation is that one can usually not decide whether a system is in the particular state $w \in \Omega$,but one can decide whether $w \in A$ or $w \notin A$.

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3- Any measure P, which gives a probability to all $A \in F$ .This probability is a evolutionary number $P(A) \in [0,1] + e_1[0,1] + e_2[0,1] + e_3[0,1]$ and $P(A) \in H$ that describes how likely it is that the event A occurs. We define the σ algebras F, here we do not need any measure.

3.3 G Theoretical probability space

Let G is a theoretical probability space.

$G = \{g^0, g^1, ..., g^k\}$

Theoretical probability is $G(A) \in [0,1]$

Theoretical probability is mathematical values of probability.

3.4 M_1 Experimental probability space

Let M_1 is a first experimental probability space.

$M_1 = \{a^0, a^1, ..., a^k\}$

Experimental probability is $M_1(A) \in [0,1]$

3.5 M_2 Experimental probability space

Let M_2 is a second experimental probability space.

$M_2 = \{b^0, b^1, ..., b^k\}$

Experimental probability is $M_2(A) \in [0,1]$

3.6 M_3 Experimental probability space

Let M_3 is a third experimental probability space.

$M_3 = \{c^0, c^1, ..., c^k\}$

Experimental probability is $M_3(A) \in [0,1]$

3.7 P Evolutionary probability space

Let P is a evolutionary probability space.

The form of a evolutionary probability number is

$\Psi = G + M_1e_1 + M_2e_2 + M_3e_3$ and $\Psi(A) = G(A) + M_1(A)e_1 + M_2(A)e_2 + M_3(A)e_3$

$$P = \{\Psi = G + M_1e_1 + M_2e_2 + M_3e_3 \in H \,|\, G, M_1, M_2, M_3 \in R\}$$

$$P = \{\Psi = G + M_1e_1 + M_2e_2 + M_3e_3 \in H \,|\, e_1^{\ 2} = e_2^{\ 2} = e_3^{\ 2} = -1, e_1e_2 = e_3, e_3e_1 = e_2, e_2e_3 = e_1, e_1e_2e_3 = -1\}$$

$$P = \{\Psi(A) = G(A) + M_1(A)e_1 + M_2(A)e_2 + M_3(A)e_3 \in H \,|\, G(A), M_1(A), M_2(A), M_3(A) \in [0,1]\}$$

Let S is a evolutionary probability set.

$$S = \{\Psi = G + M_1e_1 + M_2e_2 + M_3e_3 \in P \,|\, G, M_1, M_2, M_3 \in R\}$$

$$S = \{\Psi = G + M_1e_1 + M_2e_2 + M_3e_3 \in P \,|\, e_1^{\ 2} = e_2^{\ 2} = e_3^{\ 2} = -1, e_1e_2 = e_3, e_3e_1 = e_2, e_2e_3 = e_1, e_1e_2e_3 = -1\}$$

$$S = \{\Psi(A) = G(A) + M_1(A)e_1 + M_2(A)e_2 + M_3(A)e_3 \in P \,|\, G(A), M_1(A), M_2(A), M_3(A) \in [0,1]\}$$

Let A and B are events

$\Psi(A) = G(A) + M_1(A)e_1 + M_2(A)e_2 + M_3(A)e_3$

$\Psi(B) = G(B) + M_1(B)e_1 + M_2(B)e_2 + M_3(B)e_3$ are evolutionary probabilities.

$$(\Psi(A) + \Psi(B)) = [G(A) + G(B)] + [M_1(A) + M_1(B)]e_1 + [M_2(A) + M_2(B)]e_2 + [M_3(A) + M_3(B)]e_3$$

$$(\Psi(A) + \Psi(B)) = 1 + 1e_1 + 1e_2 + 1e_3$$

$$|\Psi(A) + \Psi(B)| = |1 + 1e_1 + 1e_2 + 1e_3|$$

$$|\Psi(A) + \Psi(B)| = \sqrt{4} = 2$$

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4 Probability definitions

4.1 G Theoretical probability

Let (Ω, F), be a measurable space.

G is mathematical probability space. We will get theoretical probability from mathematical results.

1- A map $G : F \to [0,1]$ is called probability measure if $G(\Omega)=1$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $G\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} G(A_i)$.

The triplet (Ω, F, G) is called theoretical probability space.

2- Any map $\mu : F \to [0,\infty]$ is called measure if $\mu(\varnothing)=0$ and for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3- Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,\ldots$, such that

a- $\Omega_k \in F$ for all $k = 1,2,\ldots$

b- $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c- $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d- $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

If we flip a coin, then we have either" heads" or "tails" on top, that means. Probability of head and probability of tail are $\frac{1}{2}$ to each of the two possible outcomes. If the coin is fair, then heads and tails should receive the same probability. The reason for this has to do with our intuitive notion of what a probability means.

In this sample, the two possible outcomes form the set

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$G(head) = G(tail) = \frac{1}{2}, G(h) = G(t) = \frac{1}{2}, G(head) + G(tail) = 1$$

4.2 M_1, M_2, M_3 Experimental probabilities

Let (Ω, F), be a measurable space.

M_1, M_2, M_3 are experimental probability spaces. We will get experimental or relative probability from experimental results.

1- A map $M_1 : F \to [0,1], M_2 : F \to [0,1], M_3 : F \to [0,1]$ are called probability measure if $M_1(\Omega)=1$, $M_2(\Omega)=1$ and $M_3(\Omega)=1$ for all $A_1, A_2, \ldots \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has

$$M_1\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} M_1(A_i)$$

$$M_2\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} M_2(A_i)$$

$$M_3\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} M_3(A_i)$$

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The triplet $(\Omega, F, M_1), (\Omega, F, M_2)$ and (Ω, F, M_3) are called experimental probability spaces.

2- Any map $\mu : F \to [0, \infty]$ is called measure if $\mu(\varnothing) = 0$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3- Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,...$, such that

a- $\Omega_k \in F$ for all $k = 1,2,...$

b- $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c- $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d- $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

If we toss the coin Q times and the number of heads among these Q tosses is Q_h, then relative frequency of heads is equal to $\frac{Q_h}{Q}$.Now if Q is large, then we tend to think about $\frac{Q_h}{Q}$ as being close to probability of heads. The relative frequency of tails can be written as $\frac{Q_t}{Q}$,where Q_t is the number of tails among the Q tosses, and we again think of $\frac{Q_t}{Q}$ as being close to the probability tails. Since $\frac{Q_h}{Q} + \frac{Q_t}{Q} = 1$, we see that at least intuitively, the probabilities of heads and tails should add up to one. If we make three experiments for probability, we will see that first, second and third experimental group results.

In this sample, the two possible outcomes form the set

4.3 M_1 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_1(head) = \frac{Q_{h1}}{Q_{M1}},\ M_1(tail) = \frac{Q_{t1}}{Q_{M1}},$$

$$M_1(head) + M_1(tail) = 1$$

4.4 M_2 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_2(head) = \frac{Q_{h2}}{Q_{M2}},\ M_2(tail) = \frac{Q_{t2}}{Q_{M2}},$$

$$M_2(head) + M_2(tail) = 1$$

4.5 M_3 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_3(head) = \frac{Q_{h3}}{Q_{M3}},\ M_3(tail) = \frac{Q_{t3}}{Q_{M3}},$$

$$M_3(head) + M_3(tail) = 1$$

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5 Ψ Evolutionary probability

Let (Ω, F), be a measurable space.

Ψ is an evolutionary probability space. We will get evolutionary probability from theoretical and experimental results.

1- A map $\Psi : F \to [0,1]$ is called probability measure if $\Psi(\Omega) = 1$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\Psi\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \Psi(A_i)$.

The triplet (Ω, F, Ψ) is called experimental probability space.

2- Any map $\mu : F \to [0,\infty]$ is called measure if $\mu(\varnothing) = 0$ and for all $A_1, A_2, ... \in F$ with $A_i \cap A_j = \varnothing$ for $i \neq j$ one has $\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$.

The triplet (Ω, F, μ) is called measure space.

3- Any measure space (Ω, F, μ) or a measure μ is called σ finite provided that there are $\Omega_k \subseteq \Omega, k = 1,2,...$, such that

a- $\Omega_k \in F$ for all $k = 1,2,...$

b- $\Omega_i \cap \Omega_j = \varnothing$ for $i \neq j$

c- $\Omega = \bigcup_{k=1}^{\infty} \Omega_k$

d- $\mu(\Omega_k) < \infty$

The measure space (Ω, F, μ) or the measure μ are called finite if $\mu(\Omega) < \infty$.

Let evolutionary probability is $\Psi(A) = G(A) + M(A)e_1 + N(A)e_2 + V(A)e_3$

and $\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

5.1 G Theoretical probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$G(head) = \frac{1}{2}, G(tail) = \frac{1}{2},$$

$$G(head) + G(tail) = 1,$$

5.2 M_1 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_1(head) = \frac{Q_{h1}}{Q_{M1}},\ M_1(tail) = \frac{Q_{t1}}{Q_{M1}},$$

$$M_1(head) + M_1(tail) = 1$$

5.3 M_2 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_2(head) = \frac{Q_{h2}}{Q_{M2}},\ M_2(tail) = \frac{Q_{t2}}{Q_{M2}},$$

$$M_2(head) + M_2(tail) = 1$$

5.4 M_3 Experimental probability

$\Omega = \{head, tail\}$ or $\Omega = \{h, t\}$

$$M_3(head) = \frac{Q_{h3}}{Q_{M3}},\ M_3(tail) = \frac{Q_{t3}}{Q_{M3}},$$

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$M_3(head)+M_3(tail)=1$

6 Ψ Quantum Evolutionary Probability

$\Omega=\{head,tail\}$or $\Omega=\{h,t\}$

$\Psi(A)=G(A)+M_1(A)e_1+M_2(A)e_2+M_3(A)e_3$

$\Psi(B)=G(B)+M_1(B)e_1+M_2(B)e_2+M_3(B)e_3$

$\Psi(head)=G(head)+M_1(head)e_1+M_2(head)e_2+M_3(head)e_3$

$\Psi(tail)=G(tail)+M_1(tail)e_1+M_2(tail)e_2+M_3(tail)e_3$

$$\Psi(head)=\frac{1}{2}+\frac{Q_{h1}}{Q_{M1}}e_1+\frac{Q_{h2}}{Q_{M2}}e_2+\frac{Q_{h3}}{Q_{M3}}e_3,$$

$$\Psi(tail)=\frac{1}{2}+\frac{Q_{t1}}{Q_{M1}}e_1+\frac{Q_{t2}}{Q_{M2}}e_2+\frac{Q_{t3}}{Q_{M3}}e_3$$

$\Psi(head)+\Psi(tail)=1+1e_1+1e_2+1e_3$

7 Propositions

7.1 Propositions

Definition.13

Let (Ω,F,P)be an evolutionary probability space. Then the following assertions are

1. If $A_1,A_2,...\in F$ such that $A_i\cap A_j=\varnothing$ for $i\neq j$ one has $P\left(\bigcup_{i=1}^{n}A_i\right)=\sum_{i=1}^{n}P(A_i)$

2. If $A,B\in F$ then $P(A\setminus B)=P(A)+P(A\cap B)$

3. If $B\in F$ then $P(B^c)=1-P(B)$,

4. If $A_1,A_2,...\in F$ then $P\left(\bigcup_{i=1}^{\infty}A_i\right)\leq\sum_{i=1}^{\infty}P(A_i)$

5. Continuity from below:

If $A_1,A_2,...\in F$ such that $A_1\subseteq A_2\subseteq A_3\subseteq...$,then $\lim_{n\to\infty}P(A_n)=P\left(\bigcup_{n=1}^{\infty}A_n\right)$

6. Continuity from above :

If $A_1,A_2,...\in F$ such that $A_1\supseteq A_2\supseteq A_3\supseteq...$,then $\lim_{n\to\infty}P(A_n)=P\left(\bigcap_{n=1}^{\infty}A_n\right)$

7. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

8. $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$

9. For pair wise events $A_1,A_2,...,A_n$ it is case that

$$P\left(\bigcup_{i=1}^{n}A_i\right)=\sum_{i}P(A_i)-\sum_{i<j}P(A_i\cap A_j)+\sum_{i<j<k}P(A_i\cap A_j\cap A_k)-...+(-1)^{n+1}P(A_1\cap A_2\cap...\cap A_n)$$

7.2. Independence events

Definition.14

Let (Ω,F,P)be an evolutionary probability space. The events $A_i\subseteq F,i\in K$ is an arbitrary non-empty index set, are called independent, provide that for all distinct $i_1,...,i_n\in K$ one has that $P(A_{i_1}\cap A_{i_2}\cap...A_{i_n})=P(A_{i_1})P(A_{i_2})...P(A_{i_n})$

Given $A_1,A_2,...\in F$, one can easily see that only demanding

$P(A_{1_1}\cap A_2\cap...A_n)=P(A_1)P(A_2)...P(A_n)$

7.3. Conditional probability

Definition.15

Let (Ω, F, P) be an evolutionary probability space. Suppose A and B are events in sample space Ω ,and suppose that $P(B) > 0$ .The conditional probability of A given B is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ and } A, B \in F$$

8 Operators

8.1. Product

Let

$$\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$$

$$P = \{a_0 + a_1e_1 + a_2e_2 + a_3e_3 \in H | a_0, a_1, a_2, a_3 \in R\}$$

$$P = \{a_0 + a_1e_1 + a_2e_2 + a_3e_3 \in H | e_1^2 = e_2^2 = e_3^2 = -1, e_1e_2 = e_3, e_3e_1 = e_2, e_2e_3 = e_1, e_1e_2e_3 = -1\}$$

$$\Psi_1 = a_0 + a_1e_1 + a_2e_2 + a_3e_3$$

$$\Psi_2 = b_0 + b_1e_1 + b_2e_2 + b_3e_3$$

Multiplication is generally noncommutative $\Psi_1 \times \Psi_2 \neq \Psi_2 \times \Psi_1$

$\Psi_1 \times \Psi_2$	e_1	e_2	e_3
e_1	-1	e_3	$-e_2$
e_2	$-e_3$	-1	e_1
e_3	e_2	$-e_1$	-1

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8.2. Conjugate

The conjugate of $\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ is $\overline{\Psi} = a_0 - a_1e_1 - a_2e_2 - a_3e_3$

$$\Psi = \{a_0 + a_1e_1 + a_2e_2 + a_3e_3 \in H | a_0, a_1, a_2, a_3 \in R\}$$

$$\overline{\Psi} = \{a_0 - a_1e_1 - a_2e_2 - a_3e_3 \in H | a_0, -a_1, -a_2, -a_3 \in R\}$$

8.3. Magnitude

The magnitude of $\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ is $|\Psi| = \sqrt{a_0^2 + a_1^2 + a_2^2 + a_3^2}$

8.4. Multiplicative Inverse

The multiplicative inverse of $\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ is

$$\Psi^{-1} = \frac{1}{\Psi}, \Psi \neq 0 \text{ and } \Psi^{-1} = \frac{\overline{\Psi}}{\Psi\overline{\Psi}}$$

$$\Psi^{-1} = \frac{a_0 - a_1e_1 - a_2e_2 - a_3e_3}{(a_0 + a_1e_1 + a_2e_2 + a_3e_3)(a_0 - a_1e_1 - a_2e_2 - a_3e_{31})}$$

$$\Psi^{-1} = \frac{a_0 - a_1e_1 - a_2e_2 - a_3e_3}{a_0^2 + a_1^2 + a_2^2 + a_3^2}$$

8.5 Division

Let

$\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$

$\Psi = \{a_0 + a_1e_1 + a_2e_2 + a_3e_3 \in H | a_0, a_1, a_2, a_3 \in R\}$

$\Psi_1 = a_0 + a_1e_1 + a_2e_2 + a_3e_3$

$\Psi_2 = b_0 + b_1e_1 + b_2e_2 + b_3e_3$ and $\Psi_2 \neq 0$

$\frac{\Psi_1}{\Psi_2} = d_0 + d_1e_1 + d_2e_2 + d_3e_3$

$\Psi = \{d_0 + d_1e_1 + d_2e_2 + d_3e_3 \in H | d_0, d_1, d_2, d_3 \in R\}$

The conjugates of $\Psi_1 = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ and $\Psi_2 = b_0 + b_1e_1 + b_2e_2 + b_3e_3$ are

$\overline{\Psi_1} = a_0 - a_1e_1 - a_2e_2 - a_3e_3$ and $\overline{\Psi_2} = b_0 - b_1e_1 - b_2e_2 - b_3e_3$

$$\frac{\Psi_1}{\Psi_2} = \frac{(a_0 + a_1e_1 + a_2e_2 + a_3e_3)(b_0 - b_1e_1 - b_2e_2 - b_3e_3)}{(b_0 + b_1e_1 + b_2e_2 + b_3e_3b_0 + b_1e_1)(b_0 - b_1e_1 - b_2e_2 - b_3e_3)}$$

$$\frac{\Psi_1}{\Psi_2} = \frac{(a_0 + a_1e_1 + a_2e_2 + a_3e_3)(b_0 - b_1e_1 - b_2e_2 - b_3e_3)}{b_0^2 + b_1^2 + b_2^2 + b_3^2}$$

8.6 Polar notation

Let

$\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$

$\Psi = \{a_0 + a_1e_1 + a_2e_2 + a_3e_3 \in H | a_0, a_1, a_2, a_3 \in R\}$

The magnitude of $\Psi = a_0 + a_1e_1 + a_2e_2 + a_3e_3$ is $|\Psi| = \sqrt{a_0^2 + a_1^2 + a_2^2 + a_3^2}$

$Arg(T) = \{\theta_1 + 2\pi k, \theta_2 + 2\pi k, \theta_3 + 2\pi k\}$ and

$\theta = \{0 \leq \theta_1 < 360^0, 0 \leq \theta_2 < 360^0, 0 \leq \theta_3 < 360^0 \mid \theta_1, \theta_2, \theta_3 \in R\}$

The radius set is $r = \{r_1 = \sqrt{a_0^2 + a_1^2}, r_2 = \sqrt{a_0^2 + a_2^2}, r_3 = \sqrt{a_0^2 + a_3^2} \mid r_1, r_2, r_3 \in R\}$

The polar notation is $\Psi = r_1(\cos\theta_1 + e_1\sin\theta_1)r_2(\cos\theta_2 + e_2\sin\theta_2)r_3(\cos\theta_3 + e_3\sin\theta_3)$

I developed these polar forms.

$$\cos\theta = \left\{\cos\theta_1 = \frac{a_0}{\sqrt{a_0^2 + a_1^2}}, \cos\theta_2 = \frac{a_0}{\sqrt{a_0^2 + a_2^2}}, \cos\theta_3 = \frac{a_0}{\sqrt{a_0^2 + a_3^2}} \mid \cos\theta_1, \cos\theta_2, \cos\theta_3 \in R\right\}$$

$$\sin\theta = \left\{\sin\theta_1 = \frac{a_1}{\sqrt{a_0^2 + a_1^2}}, \sin\theta_2 = \frac{a_2}{\sqrt{a_0^2 + a_2^2}}, \sin\theta_3 = \frac{a_3}{\sqrt{a_0^2 + a_3^2}} \mid \sin\theta_1, \sin\theta_2, \sin\theta_3 \in R\right\}$$

The polar notation is

$\Psi = r_1(\cos\theta_1 + e_1\sin\theta_1)r_2(\cos\theta_2 + e_2\sin\theta_2)r_3(\cos\theta_3 + e_3\sin\theta_3)$

Its conjugate is

$\overline{\Psi} = r_1(\cos\theta_1 - e_1\sin\theta_1)r_2(\cos\theta_2 - e_2\sin\theta_2)r_3(\cos\theta_3 - e_3\sin\theta_3)$

8.7 Exponential form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k, \theta_2 + 2\pi k, \theta_3 + 2\pi k\}$ and

$\theta = \{0 \leq \theta_1 < 360^0, 0 \leq \theta_2 < 360^0, 0 \leq \theta_3 < 360^0 \mid \theta_1, \theta_2, \theta_3 \in R\}$,

The radius set is $r = \{r_1 = \sqrt{a_0^2 + a_1^2}, r_2 = \sqrt{a_0^2 + a_2^2}, r_3 = \sqrt{a_0^2 + a_3^2} \mid r_1, r_2, r_3 \in R\}$

I developed these exponential forms.

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The polar form is $\Psi = r_1(\cos\theta_1 + e_1\sin\theta_1)r_2(\cos\theta_2 + e_2\sin\theta_2)r_3(\cos\theta_3 + e_3\sin\theta_3)$

The exponential form is

$$e^{e_1\theta_1 + e_2\theta_2 + e_3\theta_3} = (\cos\theta_1 + e_1\sin\theta_1)(\cos\theta_2 + e_2\sin\theta_2)(\cos\theta_3 + e_3\sin\theta_3)$$

Its conjugate is

$$e^{-e_1\theta_1 - e_2\theta_2 - e_3\theta_3} = (\cos\theta_1 - e_1\sin\theta_1)(\cos\theta_2 - e_2\sin\theta_2)(\cos\theta_3 - e_3\sin\theta_3)$$

8.8 Power form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k, \theta_2 + 2\pi k, \theta_3 + 2\pi k\}$ and

$\theta = \{0 \le \theta_1 < 360^0, 0 \le \theta_2 < 360^0, 0 \le \theta_3 < 360^0 \mid \theta_1, \theta_2, \theta_3 \in R\}$,

The radius set is $r = \left\{r_1 = \sqrt{a_0^{\ 2} + a_1^{\ 2}}, r_2 = \sqrt{a_0^{\ 2} + a_2^{\ 2}}, r_3 = \sqrt{a_0^{\ 2} + a_3^{\ 2}} \mid r_1, r_2, r_3 \in R\right\}$

I developed these power forms.

The polar form is $\Psi = r_1(\cos\theta_1 + e_1\sin\theta_1)r_2(\cos\theta_2 + e_2\sin\theta_2)r_3(\cos\theta_3 + e_3\sin\theta_3)$

The power form is from degree nth power and $n \in Z$

$$\Psi^n = r_1^n(\cos n\theta_1 + e_1\sin n\theta_1)r_2^n(\cos n\theta_2 + e_2\sin n\theta_2)r_3^n(\cos n\theta_3 + e_3\sin n\theta_3)$$

8.9 Root form

Let

$Arg(\Psi) = \{\theta_1 + 2\pi k, \theta_2 + 2\pi k, \theta_3 + 2\pi k\}$ and

$\theta = \{0 \le \theta_1 < 360^0, 0 \le \theta_2 < 360^0, 0 \le \theta_3 < 360^0 \mid \theta_1, \theta_2, \theta_3 \in R\}$,

The radius set is $r = \left\{r_1 = \sqrt{a_0^{\ 2} + a_1^{\ 2}}, r_2 = \sqrt{a_0^{\ 2} + a_2^{\ 2}}, r_3 = \sqrt{a_0^{\ 2} + a_3^{\ 2}} \mid r_1, r_2, r_3 \in R\right\}$

I developed these root forms.

The root form is from degree nth root, $k = 0,1,2,...,n-1$ and $k, n \in Z$

$$\Psi_k = \sqrt[n]{r_1}\left(\cos\frac{\theta_1 + 2k\pi}{n} + e_1\sin\frac{\theta_1 + 2k\pi}{n}\right)\sqrt[n]{r_2}\left(\cos\frac{\theta_2 + 2k\pi}{n} + e_2\sin\frac{\theta_2 + 2k\pi}{n}\right)\sqrt[n]{r_3}\left(\cos\frac{\theta_3 + 2k\pi}{n} + e_3\sin\frac{\theta_3 + 2k\pi}{n}\right)$$

Its roots are $\Psi_k = \{\Psi_0, \Psi_1, ..., \Psi_{n-1}\}$

8.10 Addition

$$\Psi_1 = a_0 + a_1e_1 + a_2e_2 + a_3e_3$$

$$\Psi_2 = b_0 + b_1e_1 + b_2e_2 + b_3e_3$$

$$\Psi_1 + \Psi_2 = (a_0 + b_0) + (a_1 + b_1)e_1 + (a_2 + b_2)e_2 + (a_3 + b_3)e_3$$

$$\Psi_1 + \Psi_2 = c_0 + c_1e_1 + c_2e_2 + c_3e_3$$

$$\Psi_1 + \Psi_2 = \{c_0 + c_1e_1 + c_2e_2 + c_3e_3 \in H \mid c_0, c_1, c_2, c_3 \in R\}$$

9 Quantum Evolutionary Probability

9.1 The physical state postulate

The state, at time t, of an isolated physical system that consists N of point particles whose positions are given by point particles, $\vec{r}_1, ..., \vec{r}_N$, is given by a well-behaved, square-integrable and normalized wave function $\Psi(\vec{r}_1, ..., \vec{r}_N, t)$ for physical states.

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The Quantum Mechanical characterization of the system's physical state is completely different from its classical counterpart, where the state is described by the actual values of and $\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N$ and $\vec{p}_1,\ldots\ldots\ldots\ldots,\vec{p}_N$ at time t.

9.2 The time evolution of Schrödinger equation for physical state postulate

The time evolution of the wave function, $\Psi(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t)$ is governed by the time dependent Schrödinger's equation for physical state.
where

$\Psi(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t)$ is probability function of the system for physical state.

$V(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t)$ is the potential energy of the system

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t)=\left(-\sum_{j=1}^{N}\frac{\hbar^2}{2m_j}\nabla^2 r_j+V(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t)\right)\Psi(\vec{r}_1,\ldots\ldots\ldots\ldots,\vec{r}_N,t) \quad (1)$$

$$\Psi=\{a_0+a_1e_1+a_2e_2+a_3e_3\in H \mid a_0,a_1,a_2,a_3\in R\}$$

$$\overline{\Psi}=\{a_0-a_1e_1-a_2e_2-a_3e_3\in H \mid a_0,-a_1,-a_2,-a_3\in R\}$$

$$|\Psi|=\sqrt{a_0{}^2+a_1{}^2+a_2{}^2+a_3{}^2}$$

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