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A REVIEW OF DIRECTION OF ARRIVAL (DOA) ESTIMATION FOR SMART ANTENNA STRUCTURE

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ABSTRACT *:In Smart Antenna System the Direction finding technique can be used to estimate the directions of the desired and interfering signals ,so that they can be separated using appropriate spatial filtering .DOA based techniques are particularly suited for wireless communication system which utilize different frequencies for uplink and downlink transmission. As well as in wireless communication, signal direction finding based on DOA estimation using an antenna array is considered another major application for antenna array processing since it enables users to be located and this information can be useful for both the, user and service provider. In this paper we compared several DOA estimation techniques in terms of spatial resolution .*

KEYWORDS: DOA, Smart Antenna, Antenna Array, Fourier Method, Barlett, Capon, MUSIC, Root-MUSIC, ESPRIT

1. INTRODUCTION

The term "Smart Antenna" generally refers to any antenna array, terminated in a sophisticated signal processor, which can adjust or adapt its own beam pattern in order to emphasize signals of interest and to minimize interfering signals. Smart Antennas generally encompass both switched beam and beam formed adaptive systems .Switched beam systems have several available fixed beam pattern. A decision is made as to which beam to access, at any given point in time, base upon the requirements of the system. Beam formed adaptive systems allow the antenna to steer the beam to any direction of interest while simultaneously nulling interfering signals[1].

In propagation channel, it was apparent that even for one source there are many possible propagation paths and angles of arrival .If several transmitters are operating simultaneously, each source potentially creates many multipath components at the receiver.

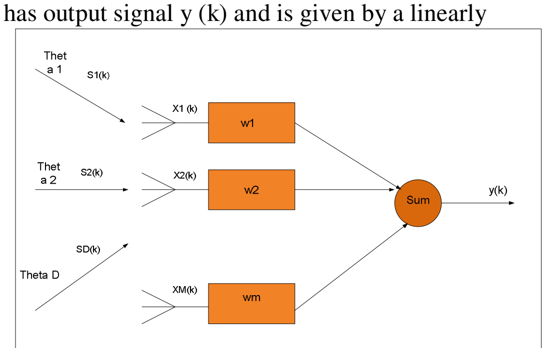
Therefore ,it is important for a receive array to be able to estimate the angle of arrival in order to decipher which emitters are present and what are their possible angular locations .

In this paper we used to compare the different DOA estimation algorithm so that, this information can be used in differentiating and finding the best algorithm for specific system.

2. DOA ESTIMATION PRINCIPLE

DOA estimation techniques can be subdivided by three main types: conventional, subspace and maximum likelihood.

The classical beamformer structure shown in fig 1, has output signal y (k) and is given by a linearly



M-element array with arriving signals

Sum of the antenna element. The fig 1 depicts a receive array with incident plane waves from various directions.

Fig1 shows D signals arriving from D directions .They are received by an array of M elements with M potential weights .Each received signal $x_m(k)$ includes additive ,zero mean ,Gaussian noise .Time is represented by the k^{th} time sample. Thus, the array output y can be given in the following form:

$$y(k) = w^T.\bar{x}(k) \quad - (1)$$

Where

$$\bar{x}(k) = [\bar{a}(\theta_1) \quad \bar{a}(\theta_2) \quad \dots \bar{a}(\theta_D)].\begin{bmatrix} s_1(k) \\ s_2(k) \\ s_D(k) \end{bmatrix} + \bar{n}(k)$$

$$= \bar{A}\,.\bar{s}(k) + \bar{n}(k) \quad - (2)$$

And

$\bar{w} = [w_1\ w_2 \dots w_M]^T$=array weights

$\bar{s}(k)$=vector of incident complex monochromatic signals at time k, $\bar{n}(k)$=noise vector at each array element m, zero mean, varianceσ_n^2, $\bar{a}(\theta_i)$ = M-element array steering vector for the θ_i direction of arrival

$\bar{A} = [\bar{a}(\theta_1)\ \ \bar{a}(\theta_2)\ \dots \bar{a}(\theta_D)]$ M*D matrix of steering vectors $\bar{a}(\theta_i)$

In order to simplify the above notation let us define the M*M array correlation matrix $\bar{R}_{xx}$ as

$$\bar{R}_{xx} = E[\bar{x}\,.\,\bar{x}^H] = E[(\bar{A}\,\bar{s} + \bar{n})(\bar{s}^H \bar{A}^H + \bar{n}^H)]$$

$$= \bar{A}^H E[\,\bar{s}.\bar{s}^H]\bar{A}^H + E[(\,\bar{n}.\bar{n}^H) \quad - (3)$$

$$= \bar{A}\bar{R}_{ss}\bar{A}^H + \bar{R}_{nn} \quad - (4)$$

Where $\bar{R}_{ss}$=D*D source correlation matrix, $\bar{R}_{nn} = \sigma_n^2 \bar{I}$=M*M noise correlation matrix, $\bar{I} = N * N$ identity matrix

The goal of DOA estimation techniques is to define a function that gives an indication of the angles of arrival based upon maxima vs. angle .This function is traditionally called the pseudo spectrum P(θ) and the units can be in energy or in watts[2] .

2. DOA ESTIMATION METHODS

2.1 Barlett DOA estimate

If the array is uniformly weighted, we can define the Barlett DOA estimate [3] as

$$P_B(\theta) = \bar{a}^H(\theta)\,\bar{R}_{xx}\bar{a}(\theta) \quad - (5)$$

The Barlett DOA estimate is the spatial version of an averaged periodogram and is a beamforming DOA estimate .Under the conditions where $\bar{s}$ represents uncorrelated monochromatic signals and there is no system noise ,Eq.(5) is equivalent to the following long hand expression:

$$P_B(\theta) = \left|\sum_{i=1}^{D}\sum_{m=1}^{M} 1.\, e^{j(m-1)kd(sin\theta - sin\theta_i)}\right|^2 \text{-(6)}$$

The periodogram is thus equivalent to the spatial finite Fourier transform of all arriving signals .This is also equivalent to adding all beam steered array factors for each angle of arrival and finding the absolute value squared.

2.2 Capon DOA estimate

The Capon DOA estimate [4, 5] is known as a minimum variance distortion less response (MVDR).It is also alternatively a maximum likelihood estimate of power arriving from one direction while all other sources are considered as interference. Thus the goal is to maximize the signal-to-interference ratio (SIR) while passing the signal of interest undistorted in phase and amplitude .The source correlation matrix $\bar{R}_{xx}$ is assumed to be diagonal .This maximized SIR is accomplished with a set of array weights, where the array weights are given by

$$\bar{w} = \frac{\bar{R}_{xx}{}^{-1}\overline{a(\theta)}}{\bar{a}^H(\theta)\bar{R}_{xx}{}^{-1}\overline{a(\theta)}} \quad - (7)$$

Where $\bar{R}_{xx}$ is the unweighted array correlation matrix.

Substituting the weights of the above equation, we can then find the pseudo spectrum is given by

$$P_C(\theta) = \frac{1}{\bar{a}^H(\theta)\bar{R}_{xx}{}^{-1}\overline{a(\theta)}} \quad - (8)$$

2.3 Linear prediction DOA estimate

The goal of linear prediction method is to minimize the prediction error between the output of the mth sensor and the actual output[6,7] .our goal is to find the weight that minimize the mean –squared prediction error .The solution of the array weights is given as

$$\overline{w_m} = \frac{\bar{R}_{xx}{}^{-1}\bar{u}_m}{\bar{u}_m{}^T\bar{R}_{xx}{}^{-1}\bar{u}_m} \quad - (9)$$

Where $\bar{u}_m$ is the Cartesian basis vector which is the mth column of the M*M identity matrix.

Upon substitution of the de array weights into the calculation of the pseudo-spectrum, it can be shown that

$$P_{LP}(\theta) = \frac{\bar{u}_m{}^T\bar{R}_{xx}{}^{-1}\bar{u}_m}{\left|\bar{u}_m{}^T\bar{R}_{xx}{}^{-1}a(\theta)\right|^2} \quad - (10)$$

The particular choice for which mth element output for prediction is random .Although the choice made can dramatically affect the final resolution.

2.4 Maximum entropy DOA estimate

The goal is to find a pseudo spectrum that maximizes the entropy function subjects to constraints [8] .So the pseudo spectrum is given by

$$P_{ME}(\theta) = \frac{1}{\bar{a}(\theta)\bar{c}_J\, \overline{c_J^H}\ \bar{a}(\theta)} \quad - (11)$$

Where $\bar{c_J}$ is the jth column of the inverse array correlation matrix ($\bar{R}_{xx}{}^{-1}$).

2.5 Pisarenko Harmonic Decomposition DOA estimate

Pisarenko Harmonic Decomposition (PHD) DOA estimate is named after Russian mathematician who devised this minimum mean-squared error approach. [9]The goal is to minimize the mean squared error of the array output under the constraint that the norm of the weight vector be equal to unity. The Eigen vector that minimizes the mean-squared error corresponds to the smallest Eigen value.

For an M=6-element array, with two arriving signals ,there will be two Eigen vectors associated with the signal and four Eigen vectors associated with the noise .The corresponding PHD pseudo spectrum is given by

$$P_{PHD}(\theta) = \frac{1}{\left|\bar{a}^H(\theta)\bar{e}_1\right|^2} \qquad - (12)$$

Where $\bar{e}_1$ is the Eigenvector associated with the smallest Eigen value λ_1 .

2.6 Min-norm DOA estimate

The Minimum-norm method was developed by Reddi [10] and Kumaresan and Tufts [11]. This method is also lucidly explained by Ermolaev and Gershman [12] .The min-norm method is only relevant for uniform linear arrays (ULA).The min-norm algorithm optimizes the weight vector by solving the optimization problem where

$$\overline{E_S^H}\,\bar{w} = 0 \qquad \text{And} \qquad \overline{w_H}\,\overline{u_1} = 1 \ . \qquad -(13)$$

Where $\bar{w}$= array weights,
$\overline{E_S}$ = subspace of D signal
Eigenvectors= [e_{M-D+1} $e_{M-D+2} \dots e_M$]
M=number of array elements
D=number of arriving signals
$\overline{u_1}$= artesian basis vector (first column of the M*M identity matrix) =Transpose [1 0 0….0]
The solution to the optimization yields the min-norm pseudo spectrum

$$P_{MN}(\theta) = \frac{(u_1^T\,\overline{E_N}\,\overline{E_N^H}\,\overline{u_1})^2}{\left|a(\theta)^H u_1^T\,\overline{E_N}\,\overline{E_N^H}\,\overline{u_1}\right|^2} \qquad - (14)$$

Where $\overline{E_N}$=subspace of M-D noise Eigenvectors= $[\bar{e}_1 \ \bar{e}_2 \dots \bar{e}_{M-D}]$
$a(\theta)^H$=array steering vector
Since the numerator term in above Eq. is constant, we can normalize the pseudo spectrum such that

$$P_{MN}(\theta) = \frac{1}{\left|\overline{a(\theta)^H E_N}\,\overline{E_N^H}\,\overline{u_1}\right|^2} \qquad - (15)$$

It should be noted that the pseudo spectrum from the min-norm method is almost identical to the PHD pseudo spectrum .The min-norm method combines all noise Eigenvectors whereas the PHD method only uses the first noise Eigenvector.

2.7 MUSIC DOA estimate

MUSIC is stands for multiple signal classification.
This approach is first posed by Schmidt [13] and is popular high resolution Eigen structure method.
MUSIC promises to provide unbiased estimates of the number of signals, the angles of arrival, and the strengths of waveforms. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix diagonal.
In this method the number of incoming signals or one must search the Eigen values to determine the number of incoming signals. If the number of the signals is D, the number of signal Eigen values and Eigen vector is D, and the number of noise Eigen values and Eigen vectors is M-D. Because MUSIC exploits the noise Eigen vector subspace, it is sometimes referred to as a “subspace method”.
The array correlation matrix assuming uncorrelated noise with equal variances.

$$\overline{R_{xx}} = \bar{A}\bar{R}_{ss}\bar{A}^H + \sigma_n^2\bar{I} \qquad - (16)$$

Now we find the Eigen values and Eigenvectors for R_{xx}. We then produce D Eigenvectors associated with the signals and M-D Eigenvectors associated with the noise .We choose the Eigenvectors associated with smallest Eigen values .For uncorrelated signals, the smallest Eigen values are equal to the variance of the noise .Now we can construct the M*(M-D) dimensional subspace spanned by the noise Eigenvectors such that

$$\overline{E_N} = [\overline{e_1} \ \overline{e_2} \dots \overline{e_{M-D}}] \qquad - (17)$$

The noise subspace Eigenvectors are orthogonal to the array steering vectors at the angles of arrival $\theta_1, \theta_2, \dots, \theta_D$ .Because of this orthogonality condition; one can show that the Euclidean distance $d^2 = \bar{a}(\theta)^H \bar{E}_N\,\overline{E_N^H}\,\bar{a}(\theta) = 0$ for each and every arrival angle$\theta_1, \theta_2, \dots, \theta_D$. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival .The MUSIC pseudo spectrum is now given as

$$P_{MU}(\theta) = \frac{1}{\left|\bar{a}(\theta)^H \bar{E}_N\,\overline{E_N^H}\,\bar{a}(\theta)\right|} \qquad - (18)$$

2.8 Root MUSIC DOA estimate

The MUSIC method in general can apply to any arbitrary array regardless of the position of the array elements [14].Root –MUSIC implies that the MUSIC algorithm is reduced to finding roots of a polynomial as opposed to merely plotting the pseudo spectrum or searching for peaks in the pseudo spectrum. Barabell [12] simplified the MUSIC algorithm for the case where the antenna is a uniform linear array.
Now recalling the MUSIC pseudo spectrum is given by $P_{MU}(\theta) = \frac{1}{\left|\bar{a}(\theta)^H \bar{E}_N\,\overline{E_N^H}\,\bar{a}(\theta)\right|}$
One can simplify the denominator expression by defining the matrix $\bar{C} = \bar{E}_N\,\overline{E_N^H}$ which is Hermitian .This leads to the root-MUSIC expression

$$P_{MU}(\theta) = \frac{1}{\left|\bar{a}(\theta)^H\,\bar{C}\ \ \bar{a}(\theta)\right|} \qquad -(19)$$

2.9 ESPRIT DOA estimate

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques and it was first proposed by Roy and Kailath [15].The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure .ESPRIT inherently assumes narrowband signals so that one knows the translational phase relationship between the multiple arrays to be used.
ESPRIT assumes multiple identical arrays called doublets .These can be separate arrays or can be composed of sub arrays of one larger array. It is

important that these arrays are displaced translational but not rotationally.

An example is shown in fig.2 where a four element linear array is composed of two identical three-element sub arrays or two doublets. These two sub arrays are translational displaced by the distance d. let us label these array 1 and array 2.

The signal induced on each of the arrays are given by

$$\overline{x_1}(k)=[\overline{a_1}(\theta_1)\ \ \overline{a_1}(\theta_2)\ ..\ \overline{a_1}(\theta_D)].\begin{bmatrix} s_1(k) \\ s_2(k) \\ s_D(k) \end{bmatrix}+\overline{n_1}(k)$$

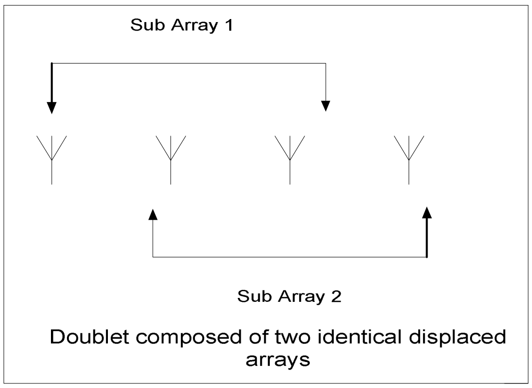


Fig 2 Doublet composed of two identical displaced arrays

$$= \bar{A}_1.\bar{s}(k) + \bar{n}_1(k) \qquad - (20)$$

And $\overline{x_2}(k) = \bar{A}_2.\bar{s}(k) + \bar{n}_2(k)$

$$= \bar{A}_1\ .\bar{\emptyset}.\bar{s}(k) + \bar{n}_2(k) \qquad - (21)$$

Where $\bar{\emptyset}$ = diag$\{e^{jkdsin\theta_1}, e^{jkdsin\theta_2}, \ldots\ldots, e^{jkdsin\theta_D}\}$

= a D*D diagonal unitary matrix with phase shifts between the doublets for each AOA

$\bar{A}_i$ = Vander monde matrix of steering vectors for sub arrays i=1,2.

The complete received signal considering the contributions of both sub arrays is given as

$$\bar{x}(k) = \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{bmatrix} = \begin{bmatrix} \bar{A}_1 \\ \bar{A}_1.\bar{\emptyset} \end{bmatrix}\ .\bar{s}(k) + \begin{bmatrix} \bar{n}_1(k) \\ \bar{n}_2(k) \end{bmatrix} \qquad - (22)$$

We can now calculate the correlation matrix for either the complete array or for the two sub arrays. The correlation matrix for the complete array is given by

$$\bar{R}_{xx} = E[\bar{x}.\bar{x}^H] = \bar{A}\bar{R}_{ss}\bar{A}^H + \sigma_n^2\bar{I} \qquad - (23)$$

Whereas the correlation matrices for the two sub arrays are given by

$$\bar{R}_{11} = E\left[\overline{x_1}.\overline{x_1}^H\right] = \bar{A}\bar{R}_{ss}\bar{A}^H + \sigma_n^2\bar{I} \qquad - (24)$$

And

$$\bar{R}_{22} = E\left[\overline{x_2}.\overline{x_2}^H\right] = \bar{A}\bar{\emptyset}\bar{R}_{ss}\ \bar{\emptyset}^H\bar{A}^H + \sigma_n^2\bar{I} \quad - (25)$$

Each of the full rank correlation matrices in above 2 Eq. has a set of Eigenvectors corresponding to the D signals present .Creating the signal subspace for the two sub arrays results in the two matrices E_1 and E_2.Creating the signal subspace for the entire array results in one signal subspace given by E_x .Because of the invariance structure of the array, E_x can be decomposed into the subspaces E_1 and E_2.

$$\bar{E}_1\bar{\Psi} = \bar{E}_2 \qquad - (26)$$

There must also exist a unique non singular transformation matrix T such that

$$\bar{E}_1 = \bar{A}\,\bar{T} \qquad - (27)$$

And

$$\bar{E}_2 = \bar{A}\,\bar{\emptyset}\,\bar{T} \qquad - (28)$$

By substituting Eq. (27) and (28) into Eq. (29) and assuming that $\bar{A}$ is of full rank, we can derive the relationship

$$T\,\bar{\Psi}\,\bar{T}^{-1} = \bar{\emptyset} \qquad - (29)$$

Thus the Eigen values of $\bar{\Psi}$ must be equal to the diagonal elements of $\bar{\emptyset}$ such that $\lambda_1 = e^{jkdsin\theta_1}$, $\lambda_2 = e^{jkdsin\theta_2}, \ldots, \lambda_D = e^{jkdsin\theta_D}$ and the columns of $\bar{T}$ must be the Eigenvectors of $\bar{\Psi}$.$\bar{\Psi}$ is a rotation operator that maps the signal subspace $\bar{E}_1$ into the signal subspace $\bar{E}_2$.

Algorithms	Advantages	Disadvantages
Barlett	Simple to implement, Robust to element perturbations, not need for a priori knowledge of specific statistical property	Depends on array size , Ability to resolve angles limited by array HPBW
Capon	Better resolution than Barlett, not need for a priori knowledge of specific statistical property	Limited by sensor noise power, competing sources are highly correlated it gives worse result
Linear prediction	Based upon prediction error	Choice can affect the final solution, depends on array element chosen
Maximum entropy	Gives the same pseudo spectra as linear prediction.	Choice of inverse correlation matrix dramatically affects the resolution

		achieved.
Pisarenko harmonic decomposition(PHD)	Peaks are indication of roots of the polynomial in the denominator.	Based on minimizing the mean squared error of the array output under the constraint that norm of weight vector be equal to 1.
Min norm	Only relevant for Uniform linear arrays(ULA)	It's Pseudo spectrum is almost same as PHD Pseudo spectrum
MUSIC	Good resolution	Lower performance than ESPRIT, sensitive to gain and phase errors, sensitive to coherent multipath
Root MUSIC	Applied only on uniform linear array(ULA)	S/N ratio is relatively low, root finding suffers a loss in accuracy
ESPRIT	High resolution ,non critical array calibration	Computationally complex limited by array geometries, Requires multiple snapshots.

6. CONCLUSION

This paper presents the comparison study of different DOA Estimation Algorithms. The goal of DOA estimation is to define a function that gives an indication of the angles of arrival based upon maxima vs. angle. In above mentioned algorithm has their specific characteristics and they actually works accordingly but for DOA estimation better resolution is the main criteria and MUSIC and ESPRIT fulfills this .Root –MUSIC and Min norm can only works upon uniform linear array. We have seen that MUSIC and ESPRIT method gives good resolution and accurate results and its accuracy of results based upon number of array elements.

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