IMAGE DENOISING USING MULTILEVEL WAVELET TRANSFORM BASED ON L^2 NORM RECOVERY

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ABSTRACT : *— This paper proposes different approaches of wavelet based image denoising methods. The search for efficient image denoising methods is still a valid challenge at the crossing of functional analysis and statistics. Wavelet algorithms are very useful tool for signal processing such as image denoising. The main application of modifying the coefficients is to remove the undesired part from the original signal and then reconstruct the signal.*

Keywords: Image Denoising, Wavelet Transform,

1. INTRODUCTION

The wavelet transform (WT) is a powerful tool of signal processing for its Multiresolution capabilities. Unlike the Fourier transform, the WT is suitable for application to non-stationary signals with transitory phenomena, whose frequency response varies in time .Usually image has noise which is not easily removed. There are many algorithms and techniques which have been developed to eliminate noise. But wavelet transform is very successful to overcome noise in data. There are two main types of wavelet transform, Continuous Wavelet Transform and Discrete Wavelet Transform. The denoising of a natural image corrupted by Gaussian noise is a classic problem in signal processing. The Wavelet Transform has become an important tool for this problem due to its energy compaction property. Indeed, wavelets provide a framework for signal decomposition in the form of a sequence of signals known as approximation signals with decreasing resolution supplemented by a sequence of additional touches called details. Denoising or estimation of functions, involves reconstructing the signal from the desired part of the original signal and reduction or removal of the undesired part of the original signal. The methods based on wavelet representations yield very simple algorithms that are often more powerful and easy to work with than traditional methods of function estimation. It consists of decomposing the observed signal into wavelets and using thresholds to select the coefficients, from which a signal is synthesized. Image denoising algorithm consists of few steps; consider an input signal x(t)and noisy signal n(t). Add these components to get noisy data y (t) i.e.

$$Y(t) = x(t) + n(t) \quad \text{................. (1)}$$

Here n(t) is white Gaussian noise. Then apply the w(t) (wavelet transform)

$$Y(t) \longrightarrow w(t) \quad \text{.....(2)}$$

Modify the wavelet coefficient w(t) using different threshold algorithm and take inverse wavelet transform to get denoising image x(t).

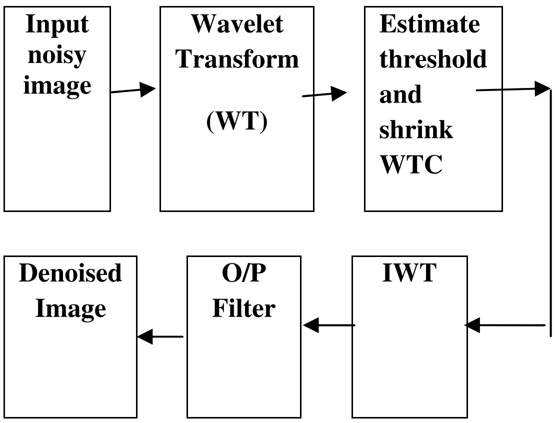

Fig.1: Block diagram of Image denoising using wavelet transform.

Image quality is expressed using L^2 norm recovery and compression score.

2: WAVELET TRANSFORM

The continuous wavelet transform was developed as an alternative approach to the short time Fourier transforms to overcome the resolution problem. The wavelet analysis is done in a similar way to the STFT analysis, in the sense that the signal is multiplied with a function, similar to the window function in the STFT, and the transform is computed separately for different segments of the time-domain signal.

$$\mathrm{CWT}_x^{\psi}(\tau, s) = \Psi_x^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t) \bullet \psi^* \left(\frac{t-\tau}{s} \right) dt$$

.... (3)

The wavelet expansion set is not unique. A wavelet system is a set of building blocks to construct or represents a signal or function. It is a two dimensional expansion set, usually a basis, for some class one or higher dimensional signals. The wavelet expansion gives a time frequency localization of the signal. Wavelet systems are generated from single scaling function by scaling and translation. A set of scaling function in terms of integer translates of the basic scaling function by

$$\mathbf{\Phi_{(s,l)}(x)=2^{s/2}\Phi(2^{-s/2}x\text{-}l),\ \Phi \in L^2} \text{.............(4)}$$

The subspaces of $L^2(R)$ spanned by these functions. A two dimensional function is generated from the basic scaling function by scaling and translation by The variables *s* and *l* are integers that scale and dilate the mother function Φ to generate Wavelets, such as a Daubechies wavelet family. The scale index *s* indicates the wavelet's width, and the location index *l* gives its position. Notice that the mother functions are rescaled, or "dilated" by powers of two, and translated by integers. What makes wavelet bases especially interesting is the self-similarity caused by the scales and dilations.

$$V_j = \overline{\mathrm{Span}}_k \{\varphi_k(2^j t)\} = V_j = \overline{\mathrm{Span}}_k \{\varphi_{j,k}(t)\} \text{...(5)}$$

for all integer. The multiresolution analysis expressed in terms of the nesting of spanned spaces as

$$\ldots\ldots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \cdots \subset L^2 \text{ .... (6)}$$

The spaces that contain high resolution signals will contain those of lower resolution also. The spaces should satisfy natural scaling condition $f(t) \in v_j \Leftrightarrow f(2t) \in v_{j+1}$ which ensures elements in space are simply scaled version of the next space. The nesting of the spans of $\varphi(2^j t - k)$ denoted by v_j i.e. $\varphi(t)$ is in v_0, it is also in v_1, the space spanned by $\varphi(2t)$. This $\varphi(t)$ can be expressed in weighted sum of shifted $\varphi(2t)$ as

$$\boldsymbol{\Phi(t) = \sum_n h(n)\sqrt{2\Phi(2t-n)},\ n \epsilon z \text{...(7)}}$$

Where the h(n) is scaling function. The factor used for normalization of the scaling function. The important feature of signal expressed in terms of wavelet function $\Psi_{j,k}(t)$not in scaling function $\varphi_{j,k}(t)$. The orthogonal complement of v_j in v_{j+1} is defined asW_j, we require,

$$[\Phi_{j,k}(t)\ \Psi_{j,l}(t)] = \int \Phi_{j,k}(t)\ \Psi_{j,l}(t)\ d(t)..(8)$$

3. DECOMPOSITION ANDMULTIRESOLUTION

An image at a given resolution can be divided into coarser approximations at a lower resolution. Suppose the original image has a resolution *rj* and its lower resolution approximation image has resolution *rj*−1. Then, the details missing in the lower resolution representation are given by the difference between the original image and the approximation image. At coarser resolutions, only the large objects are visible and the viewer gets only a rough idea of the image context. The original image can be reconstructed as successive details are added to the approximations and the finer details of the image become visible. of multiresolution are given below

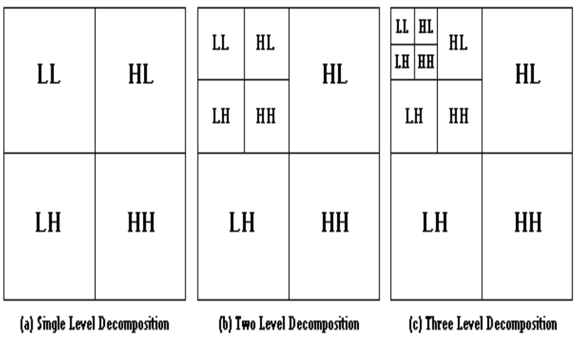

Fig.2 Different level Decomposition

The basic operations for calculating the DWT is convolving the samples of the input with the low-pass and high-pass filters of the wavelet and downsampling the output. Daubechies realized that this method of computing the DWT involved a lot of redundancy as half of the samples computed are discarded by the downsampling operator. They proposed the lifting scheme, which calculates the DWT in approximately half the number of steps required to calculate the DWT through filtering and downsampling.

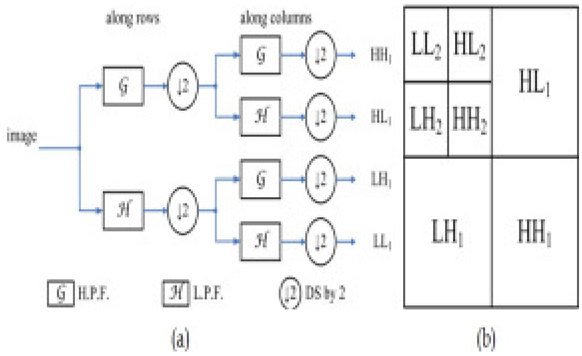

Fig.3 Two level decomposition with block diagram

4. THRESHOLDING

Estimating a signal that is corrupted by additive noise has been of interest to many researchers for practical as well as theoretical reasons. The problem is to recover the original signal from the noisy data. We want the recovered signal to be as close as possible to the original signal, retaining most of its important properties (e.g. smoothness). Traditional denoising schemes are based on linear methods, where the most common choice is the Wiener filtering. Recently, nonlinear methods, especially those based on wavelets have become increasingly popular.

Let us consider a signal *xi*, which is corrupted by additive Gaussian random noise

zi ~ *N*(0, *σ*2) as follows.

$$yi = xi + zi\ (i = 0, 1, \ldots, N - 1).......(9)$$

We first take the wavelet transformation of the noisy signal and pass it through the thresholding function *T*(·). The output is then inverse wavelet transformed to obtain the estimate ˜**x**. The most common choices for *T*(·) are the hard-thresholding function and the soft-thresholding function (which is also known as the wavelet shrinkage function).

The hard-thresholding function chooses all wavelet coefficients that are greater than the given threshold λ and sets the others to zero.

$$fh(x) = \begin{cases} x if x \geq \lambda \\ 0 \; otherwise \end{cases} \qquad (10)$$

The threshold λ is chosen according to the signal energy and the noise variance $\sigma 2$. If a wavelet coefficient is greater than λ, we assume that it is significant and attribute it to the original signal. Otherwise, we consider it to be due to the additive noise and discard the value. The soft-thresholding function has a somewhat different rule from the hard-thresholding function. It shrinks the wavelet coefficients by λ towards zero, which is the reason why it is also called the wavelet shrinkage function

$$fs(x) = \begin{cases} x - \lambda if x \geq \lambda \\ 0 \qquad if x < \lambda \\ x + \lambda if x \geq \lambda \end{cases} \text{........ (10)}$$

Note that the hard-thresholding function is discontinuous at $|x| = \lambda$. Due to this discontinuity at the threshold, the hardthresholding function is known to yield abrupt artifacts in the denoised signal, especially when the noise level is significant

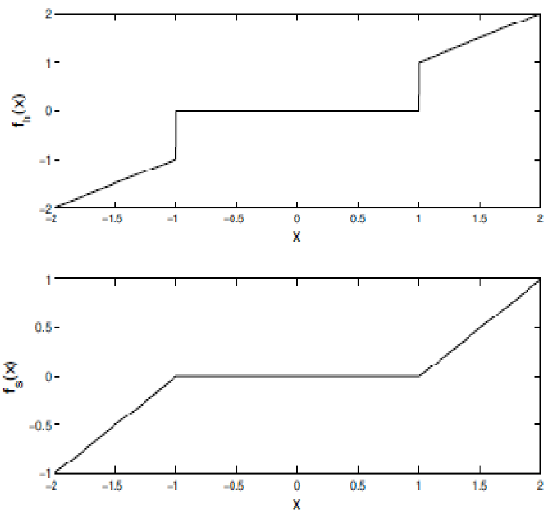

Fig.4. Soft and Hard thresholding

Estimation error than the optimal hard-thresholding estimator. For this reason, soft-thresholding is generally preferred to hard- thresholding. However, for some class of signals, we could see that hardthresholding results in superior estimates to that of soft- thresholding despite some of its disadvantages.

5. RESULT

Here Matlab software is used for developing the required simulation software to get the results shown below.

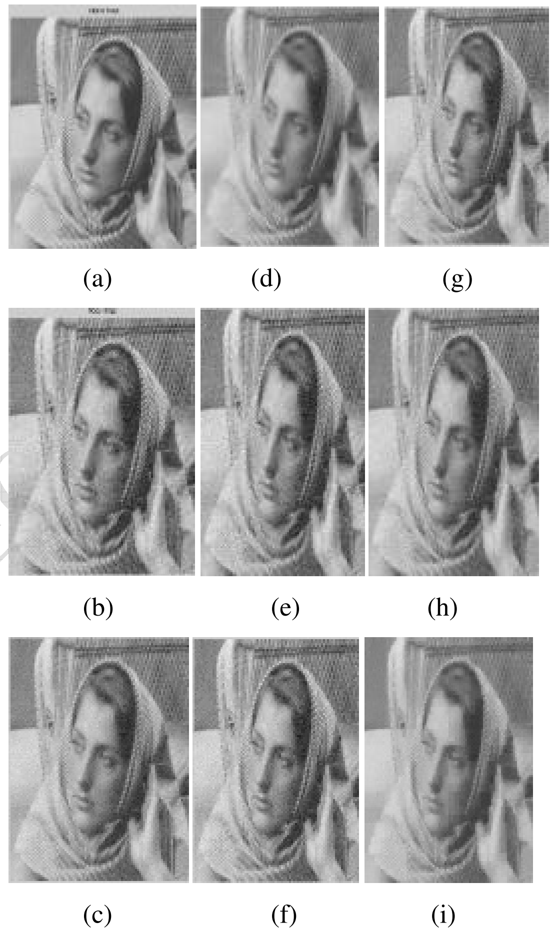

Fig.5. (a)Original image(b) noisy image. Reconstructed,images(c)With,thr=10,'s','sym4'(d) with,thr=30,'s','sym4'(e)thr=10,'h','sym4'(f)thr=30,'h','sym4'(g)thr=10,'s','haar'(h)thr=10,'h','haar' (i) thr=30,'h','haar' [Ref. image Woman from Matlab]

Wavelet	**Sym4**

Decomposition level		5			2		
Threshold value		10	20	30	10	20	30
Soft Thresholding	L^2_norm Recovery (%)	29.3541	54.0617	71.8528	27.8971	51.6823	68.724
	Compression score (%)	99.1055	98.5126	98.1179	98.1198	96.9562	96.2527
Hard Thresholding	L^2_norm Recovery (%)	29.3541	54.0617	71.8528	27.8971	51.6823	68.724
	Compression score (%)	99.9799	99.8605	99.6264	99.9509	99.6569	99.0832
Wavelet		Haar					
Decomposition level		5			2		
Threshold value		10	20	30	10	20	30
Soft Thresholding	L^2_norm Recovery (%)	27.665	51.6556	69.5282	26.651	49.6582	66.7252
	Compression score (%)	97.4402	95.7303	94.5926	97.8644	96.5184	95.6958
Hard Thresholding	L^2_norm Recovery (%)	27.655	51.6556	69.5282	26.651	49.6582	66.7252
	Compression score (%)	99.9473	99.6284	98.9761	99.9493	99.6436	99.0208

Table: 1 L^2_norm Recovery and compression score with difference threshold value

6. CONCLUSION

This technique is computationally faster and gives better results. Both for hard and soft thresholding, as threshold value is increased, L2 norm recovery is increased but compression score is decreased. We get better results using sym4 than Haar.L2 norm recovery is more if one uses more number of decomposition levels. Soft thresholding is better for denoising and hard thresholding is better for compression.

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