SIMPLE ROBUST POWER FLOW METHOD FOR RADIAL DISTRIBUTION SYSTEMS

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ABSTRACT: *A simple power flow method which is computationally efficient is reported that is robust and insensitive to Network topology and Types of Load model. IEEE 33-bus system is used as a test system to demonstrate the effectiveness of the proposed work. The power flow results for constant power load obtained for 33-bus system have been verified with the results published with other existing method and found to be in exact agreement. The convergence characteristics are analyzed for various system parameters and loading conditions. The proposed distributed power flow method is superior in computational Efficiency when compared with six other existing methods for 33-node network.*

KEYWORDS: *Power flow program, radial distribution system, load modeling*

1. Introduction

A distribution system represents the final link between the bulk power system and the consumers, therefore it is crucial to have an accurate analysis for such systems [1]. A robust and reliable radial power flow analysis represents an essential requirement for many distribution management systems applications, such as network optimization, voltage control, state estimation, service restoration, etc [2]. The topological properties of the distribution network can be exploited to devise dedicated power-flow techniques. These falls broadly under two categories-the loop based methods [3,4] and branch-based methods [5-12].

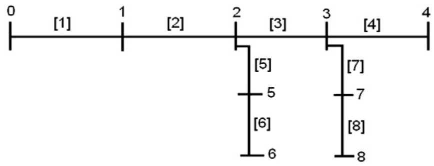
The analysis methods more frequently adopted in radial distribution systems, are iterative backward/forward methodology [13]. Baran and Wu [5] developed the power flow solution by iterative solution of the three fundamental power flow equations representing the active power, reactive power and voltage, instead of two equations. Kersting's [6] power flow method is found out to be fastest but suffers from convergence problem because it makes an assumption of pure ladder network which is seldom the case. Shirmohammadi's [7] compensation based method uses direct application of KCL and KVL and is found to be excellent for weakly meshed networks but falters for radial distribution networks. Renato [8] proposed an electrical equivalent for each node by summing all loads of network fed through the node but it solves the network for bus voltage magnitude only. Jasmon and Lee [9] reduced the whole distribution network into a single line equivalent. Goswami and Basu [10] proposed a direct solution technique that provides excellent convergence characteristics but the limitation is that no node in the network is the junction of more than three branches. Das et.al. [11] proposed a power flow technique by calculating the total active and reactive power fed through any node.

In this paper, a simple and an efficient method for solve radial distribution system is proposed. The power flow program is computationally efficient with good convergence property and is independent of load model, nos. of laterals and sub-laterals, nos. of buses and R/X ratio of conductors. Loads, in the present formulation, have been represented as constant complex power as these are the types of loads that create the most stress in the system. IEEE 33-bus system has been successfully solved. The results obtained are presented in the section V.

2. Mathematical Modeling

It is assumed that the three-phase radial distribution systemic balanced and so can be represented by their equivalent single line diagram. Figure 1 shows a single line diagram of a radial distribution system with nodes and branch numbering scheme. The electrical equivalent of one branch of Figure 1 is shown in Figure 2.

Fig 1. Single-line diagram of radial distribution network



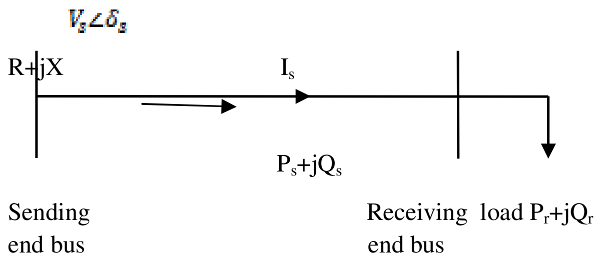


Fig. 2. Electrical equivalent of one branch of Fig. 1.

2.1 Line and Model

The length of most of distribution line sections is short, line shunt capacitance is negligible at the distribution voltage levels and can be neglected in most practical cases.In the present formulation, loads have been represented as constant complex power i.e. load power is independent of voltage.

The different methods for load flow equations of distribution systems use recursive equations in several forms considering either sending or receiving end power.

From Figure 2, $$I_s = \frac{V_s\angle\delta_s - V_r\angle\delta_r}{R+jX} \quad (1)$$

where R and X represents the resistance and reactance of the branch connecting the sending-end node and receiving-end node, $V_s$and $V_r$is the voltage of sending-end and receiving-end node respectively, and δ_s and δ_r is the voltage angle of the sending-end and receiving-end node respectively.

Complex power is given by

$$S = P_r + jQ_r = V_r I_s^* \quad (2)$$

where P_r and Q_r represents the total real and reactive power load respectively.
Taking Complex conjugate of both sides,

$$P_r - jQ_r = V_r^* I_s \quad (3)$$

From Eqn.1 and Eqn. 3, we get receiving end voltage equation in terms of receiving end powers

$$V_r = \left[\sqrt{\left(RP_r + XQ_r - \tfrac{1}{2}V_s^2\right)^2 - (R^2 + X^2)(P_r^2 + Q_r^2)} - \left(RP_r + XQ_r - \tfrac{1}{2}V_s^2\right)\right]^{\frac{1}{2}} \quad (4)$$

From Eqn. 2 and Eqn. 3,

($P_r + jQ_r)(P_r - jQ_r) = ()$

Branch current is given by (5)

Real Power loss (P_{loss}) in a line connecting a sending end bus and receiving end bus is given by

$$P_{loss} = I_s^2 R = R\left(\frac{P_r^2 + Q_r^2}{V_r^2}\right) \quad (6)$$

Reactive Power loss (Q_{loss}) in a line connecting a sending end bus and receiving end busis given by

$$Q_{loss} = I_s^2 X = X\left(\frac{P_r^2 + Q_r^2}{V_r^2}\right) \quad (7)$$

where in Eqn. 6 and Eqn. 7 is given by Eqn. 4

2.2 Node Determination beyond Branch

The method used to determine the number of nodes beyond each branch and total number of nodes is based on [14, 15]. For the single-line diagram of Figure 1, the branch-to-node incidence matrix is

$$BN = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

The node-to-branch incidence matrix is given by the inverse of the branch-to-node incidence matrix

$$NB = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

The column identifies the branches and row identifies the node. For the branch 2 (column 2), the non-zero elements correspond to the rows 2, 3, 4, 5, 6, 7 and 8. Therefore, after branch 2 we count nodes 2, 3, 4, 5, 6, 7 and 8.

From matrix NB, a branch matrix BR is formedsuch that its non-zero elements appear first in each row.

$$BR = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 \\ 3 & 4 & 7 & 8 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The row identifies the branches. N(i) is the total number of nodes after the branch i.For each branch i, the non-zero values BR(i,j), for j varying from 1 to N(i), are the nodes belonging to the considered branch. N(i) equal to one means that the node (or branch) is a terminal node (or branch). The feeder connectivity of the single-line diagram of Figure 1 is shown in Table 1.

TableI: Feeder Connectivity of Fig. 1

Branch No.	Sending-end node	Receiving-end node	Nodes beyond branch	Total no. of nodes beyond branch
1	0	1	1,2,3,4,5,6,7,8	8
2	1	2	2,3,4,5,6,7,8	7
3	2	3	3,4,7,8	4
4	3	4	4	1
5	2	5	5,6	2
6	5	6	6	1
7	3	7	7,8	2
8	7	8	8	1

3. Power Flow Solution Methodology

The voltage magnitude at each node of radial distribution feeder is determined by the following solution steps

a) Read System Topology, Line and Load Data

b) Identify the nodes beyond each branch and total number of nodes.

c) Voltages at all the buses including the source node are initialised to a flat start of 1.0 p.u. Active and Reactive power losses are set to zero. Set iteration count = 0.

d) Compute the total real power load (P_r) fed through each node which is equal to sum of the active power of all the loads beyond that node, including that node plus the sum of the active power losses of the branches beyond that node.

For example, from the matrix BR, below the branch 2 (row 2) we find the nodes 2, 3, 4, 5, 6, 7 and 8. The total number of nodes located below this branch is N(2)=7. Thus, the real power fed through the receiving-end of the branch 2 will be given by

$$P_2 = P_{load6}+ P_{load5}+P_{load2} + P_{load3} + P_{load8}+ P_{load7}+ P_{load4} + P_{loss6}+ P_{loss5}+ P_{loss3}+ P_{loss8} + P_{loss7} + P_{loss4}$$

e) Compute the total reactive power load (Q_r) fed through each node which is equal to sum of the reactive power of all the loads beyond that node, including that node plus the sum of the reactive power losses of the branches beyond that node.

f)

For example, from the matrix BR, below the branch 3 (row 3) we find the nodes 3, 4, 7 and 8. The total number of nodes located below this branch is N(3) =4. Thus the reactive power fed through the receiving-end of the branch 3 will be given by

$$Q_3 = Q_{load8}+ Q_{load7}+Q_{load3} + Q_{load4} + Q_{loss8}+ Q_{loss7}+ Q_{loss4}$$

g) Calculate the voltage magnitude at each node using equation (4), starting with the node nearest to source node.

h) Repeat steps d-f until the algorithm converges. The convergence criterion is the difference between node voltages of two subsequent iterations for all the nodes is less than the tolerance ε.

$$|V_r(i+1) - V_r(i)| < 0.0001 \text{ p.u.}$$

i) Compute the feeder current through each branch using Equation 5

j) Compute the real and reactive power loss in each branch using Equation 6 and 7.

4. Case Study

To validate and demonstrate the effectiveness of the power flow program, it has been tested for its robustness, accuracy and speed by implementing on many feeders such as IEEE 15-bus, 23-bus, 31-bus system, IEEE 33-bus system, 50-bus and IEEE 69-bus distribution system.

However, due to space constraints, numerical results of 33-bus systems are presented. The single line diagram of 33-node radial distribution system is shown in Figure3. The line and load data for IEEE 33 bus system is given in [16]. The base case power flow results for the same are tabulated in Table 8.

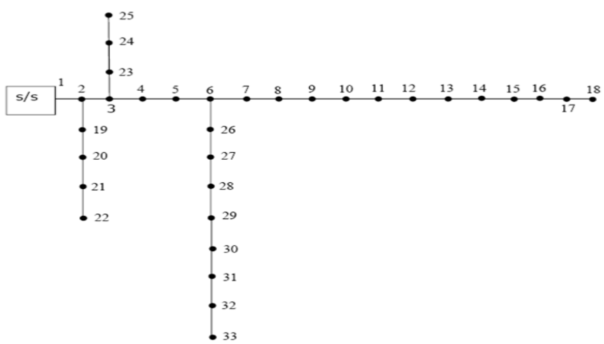


Fig 3.33-nodes single line diagram

5. Simulation Results

The proposed algorithm has been implemented in MATLAB 7.10 and run on Intel's i5 processor with Windows 7 operating system and clock of 1.6 GHz. A tolerance of p.u. on voltage magnitude is used as a convergence criterion. The summary of results of 33-bus system are shown in Table 2.The base voltage is taken as 12.66 KV. The base power for 33-bus system is 10 kVA. The power flow results obtained for 33-bus system have been verified with the results published in [16] and found to be in exact agreement.

Table 2: Test Results

No. of nodes	Iterations	Power losses		V_{min}= V_{18}p. u.	CPU Time (sec)
		KW	KVAR		
33	4	210.97 (5.67%)	143.11 (6.22%)	0.903	0.06

The convergence characteristics is also analysed for various system parameters and loading conditions [17]. The number of iterations required for different values of tolerance on voltage magnitude as a convergence criterion for 33-bus system is shown in Table 3.

Table 3: Effect of Tolerance on Convergence

Tolerance	Iterations for convergence
10^{-4}	4
10^{-5}	5
10^{-6}	6
10^{-7}	7

The effect of system loading on convergence for a substation voltage of 1.0 is calculated by increasing real and reactive power load gradually at all the busses and is shown in Table 4. The critical loading factor for different sending end source voltage from the substations for 33-bus system beyond which the system collapses is shown in Table 5.

Table 4: Effect of system loading on convergence at substation voltage of 1.0p.u.

System loading	Iterations for convergence
P+jQ	4
2 (P+jQ)	6
3 (P+jQ)	11
3.21 (P+jQ)	14
3.4 (P+jQ)	40

Table 5: Critical Loading Factor for Different Substation Voltages

Substation voltage	33-bus
1.0	3.4
1.025	3.58
1.05	3.75

To create ill conditioning, R is increased in steps of 0.5 times without changing the reactances and checked for convergence. From Table 6, it is observed that with the increase in R/X ratio, convergence deteriorates.

Table 6: Effect of Different R/X Ratio on Convergence

	Iterations for convergence
R+jX	4
1.5R+jX	5
2R+jX	5
2.5R+jX	6
3R+jX	7
3.5R+jX	9
3.6R+jX	10
4R+jX	13
4.32R+jX	47

The proposed method is also compared with six other existing methods. Table 7shows the CPU time and number of iterations of all six examples. All these six examples were simulated on a MATLAB 7.10 and run on Intel's i5 processor with Windows 7 operating system and clock of 1.6 GHz. From Table 7, it is observed that the proposed distributed power flow method is superior in terms of time taken for execution when compared with previously reported.

Table 7: Performormance Comparison of Convergence Speed Of Proposed Method With Other Existing Methods

	33-node	
	CPU time (sec)	Iterations
Proposed Method	0.06	4
Ghosh & Das [18]	0.09	3
Baran & Wu [19]	0.13	3
Chiang [20]	0.11	3
Jasmon & Lee [9]	0.13	3
Renato [8]	0.14	4
Kersting [6]	0.16	4

Table 8: Base Case Power Flow Results for 33 Bus System

Node No.	Voltage p.u.	Node No.	Voltage p.u.
1	1.0	18	0.90378
2	0.99701	19	0.99649
3	0.98288	20	0.99291
4	0.97537	21	0.9922
5	0.96795	22	0.99157
6	0.94947	23	0.9793
7	0.94595	24	0.97263
8	0.93229	25	0.9693
9	0.92596	26	0.94754
10	0.9201	27	0.94498
11	0.91923	28	0.93353
12	0.91772	29	0.92532
13	0.91154	30	0.92176
14	0.90925	31	0.9176
15	0.90783	32	0.91668
16	0.90644	33	0.9164
17	0.9044		

6. Conclusions

A simple and robust load-flow technique has been proposed for solving radial distribution systems. The effectiveness of the power flow program has been validated on 33-node radial distribution system and is found useful for planning and operation of automated radial distribution systems. The method has good and fast convergence characteristics and is compared with six other existing methods on 33-node radial distribution systems.

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