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SELECTION OF NEW CAR USING FUZZY AHP APPROACH

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ABSTRACT*-An analytical way to reach the best decision is more preferable in many business platforms. When variables are quantitative and number of criteria is not high, then one can use several analysis tools and make his/her decision and solve the problem. The conventional AHP seems to insufficient and imprecise to capture the right judgments of decision-maker(s). Fuzzy AHP is a synthetic extension of classical AHP method when the fuzziness of the decision makers is considered. In this paper, discus fuzzy AHP approach with case study of selection of new car model.*

Keywords: Analytical Hierarchy Process, Fuzzy set, Selection Criteria, Fuzzy Analytical Hierarchy Process

I. Introduction

Human lives are the sum of their decisions-whether in business or in personal spheres. In daily lives, people often have to make decisions. *"When* decision is made" is important as "*what* decided". Everyday life and history are full of lessons that can help people recognize that critical moment. People learn by trying and by example. Deciding too quickly can be hazardous; delaying too long can mean missed opportunities. In the end, it is crucial that people make up their mind. What people need is a systematic and comprehensive approach to decision making.

Many methods for estimating the preference values from the pair wise comparison matrix have been proposed and their effectiveness comparatively evaluated. Some of the proposed estimating methods presume interval-scaled preference values. But most ofthe estimating methods proposed and studied are within the paradigm of the analytichierarchy process that presumes ratio-scaled preference values. Analytical Hierarchy
Process (AHP) is one of the best ways for deciding among the complex criteria structure in different levels. Fuzzy AHP is a synthetic extension of classical AHP method when the fuzziness of the decision maker is considered. This paper shows how to work fuzzy AHP approach for making decision to select car

II. AHPand Fuzzy – AHP

A. Classical AHP

AHP is a method for ranking decision alternatives and selecting the best one when the decision maker has multiple criteria. In AHP, preferences between alternatives are determined by making pair wise comparisons. In a pair wisecomparison, the decision maker examines two alternatives byconsidering one criterion and indicates a preference. Thesecomparisons are made using a preference scale, which assigns numerical values to different levels of preference The standard preference scale used for AHP is 1-9 scale which lies between "equal importance's" to "extreme importance". If the importance of one factor with respect to a second is given, then the importance of the second factor with respect to the first is the reciprocal. Ratio scale and the use of verbal comparisons are used for weighting ofquantifiable and non-quantifiable elements. Each comparison is transformed into anumerical value of the Saaty's discrete 9-valuescale (see Table 1).

The application of the AHP to the complex problem usually involves four major steps.

1. Break down the complex problem into a number of small constituent elements and then structure the elements in a hierarchical form.
2. Make a series of pair wise comparisons among the elements according to a ratio scale.
3. Use the eigenvalue method to estimate the relative weights of the elements.
4. Aggregate these relative weights and synthesize them for the final measurement ofgiven decision alternatives.

The overall weight coefficient, with respect to the goal for each decision alternative is then obtained. The alternative with the highest weight coefficient value should be taken as the best alternative.

III.Fuzzy Analytical Hierarchical process (FAHP)

In the classic method of AHP which was introduced by Saaty (1980), each expert have to do the pair comparisons of characteristics in the same level of hierarchical structure with using exact numbers and ratios. A large number of researchers believe that as there is a kind of uncertainty in experts 'opinions, when doing pair comparisons and assigning ratio to them, the decision making would be imprecise andunreliable(Leung and

Cao, 2000). Leung and Cao (2000) believe that as in the traditional hierarchical structuremethods, each expert is asked, based on his/her understanding, to assign an exact ratio to each pair comparison, these methods are of low precision. In fact, they believe that understanding of each person from the significance ofa phenomenon than another cannot be stated in the form of a crisp number but it can be only stated in the form ofan interval of numbers. Thus, hierarchical analysis in the fuzzy state is able to simulate the decision makingprocess in human mind better than the traditional ones. The hierarchical model should be able to break the existing complex decision problem into manageable components of different layers/levels. The fuzzy AHP, is the fuzzy extension of AHP was developed to solve the hierarchical fuzzy problems.According to the responses on the question form, the correspondingtriangular fuzzy values for the linguistic variables are placed and for a particularlevel on the hierarchy the pair wise comparison matrix is constructed.

A. Establishing triangular fuzzy numbers

In the Saaty (1980) model, the geometric mean represents the expert consensus and is the most widely used in practical applications. Here, the geometric mean models the triangular fuzzy numbers. Zadeh (1965) introduced the fuzzy set theory to deal with the uncertainty due toimprecision and vagueness. A major contribution of fuzzy set theory is its capability ofrepresenting vague data. Mathematical operations and programming may also apply to a fuzzydomain (Lious and Wang 1992). A fuzzy set is a class of objects with a graded continuum of membership. Such a set is characterized by amembership function, which assigns to each object membership grade between zero and one. Figure 1.depicts a triangular fuzzy number
Since each number in the pair-wise comparison matrixrepresents the subjective opinionof decision makers. A TFN is denoted simply as (L, M, U). The parameters L, M and U denote the smallest possible value, the most promising value and the largest possible value, respectively, that describes a fuzzy event..The triangular fuzzy numbers are established as follows:

$$\tilde{u}_{ij} = \left(L_{ij}, M_{ij}, U_{ij}\right),$$

$L_{ij} \leq M_{ij}, \leq U_{ij} and L_{ij}, M_{ij}, U_{ij} \in [1/9,9]$(1)
we proposed the three fuzzy parameters torepresent conventional Saaty's AHP 1 – 9 relative importance scale given bymeans of the following equations$\tilde{1} \equiv (1,1,1)$, $\tilde{x} \equiv (x\text{-}1, x, x+1)$ $\forall x = 2,3,..,8$ and$\tilde{9} \equiv (9, 9, 9)$.Fuzzy scale has been presented in Table 2.

Table 1 Saaty's discrete 9-value scale

Numerical value	Linguistic definition
1	Equal importance
3	Weak importance of one over Another
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate judgments between two adjacent judgments

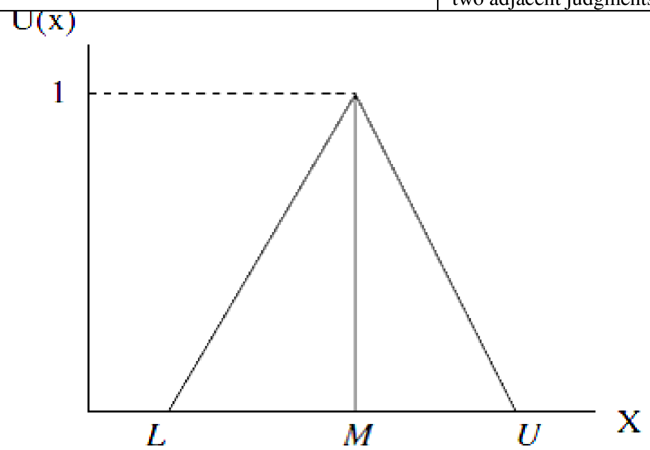

Fig. 1 Triangular fuzzy numbers

B. Establishing fuzzy pair-wise comparison matrix and defuzzification

$$\tilde{A} = [\tilde{a}_{ij}] = \begin{array}{c} C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \overset{\begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array}}{\begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \frac{1}{\tilde{a}_{12}} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\tilde{a}_{1n}} & \frac{1}{\tilde{a}_{2n}} & \cdots & 1 \end{bmatrix}}.$$

In matrix $\tilde{a}_{12}$denotes a triangular fuzzy number for the relative importance of two criteriaC_1 and C_2.various defuzzication methods are available, and the method adopted herein was derived from Liou and Wang (1992). As shown in formulae this method can clearly express fuzzy perception

Table 2.Conversion of linguistic variables to triangular fuzzy numbers

Scale of respected fuzzy number	Fuzzy number	Linguistic variable
(1 ,1 ,1)	$\tilde{1}$	Identical
(4, 3, 2)	$\tilde{3}$	a little more important
(6,5, 4)	$\tilde{5}$	More important
(8,7,6)	$\tilde{7}$	Much more important
(9,9, 9)	$\tilde{9}$	Strictly more important
x-1,x,x+1	$\tilde{2},\tilde{4},\tilde{6},\tilde{8}$	A value between 2 levels
$\left(\frac{1}{x+1},\frac{1}{x},\frac{1}{x-1}\right)$	$1/\tilde{x}$	Inverse triangular numbers

$$g_{\alpha,\beta}(\tilde{a}_{i,j}) = [\beta, f_\alpha(l_{i,j}) + (1-\beta). f_\alpha(u_{i,j})] \quad (2)$$

$$g_{\alpha,\beta}(\tilde{a}_{j,i}) = 1/g_{\alpha,\beta} \quad (3)$$

Where $0 \le \alpha \le 1, 0 \le \beta \le 1$

$$f_\alpha(l_{i,j}) = (m_{i.j} - l_{i,j})\alpha + l_{i,j}$$

$$f_\alpha(u_{i,j}) = u_{i,j} - (u_{i,j} - m_{i,j})\alpha$$

Because this method can explicitly display the preferences (α) and risk tolerance (β) of decision makers, decision makers can more thoroughly understand the risks they face in different circumstances.
αcan be viewed as a stable or fluctuating condition. The range of uncertainty is greatest when $\alpha = 0$ Meanwhile, the decision making environment stabilizes as α increases; simultaneously, the variance in decision making decreases. Additionally, α can be any number between 0 and 1, and analysis is normally set as the following 10 numbers, 0.1, 0.2,...,1 for uncertainty emulation. Further, $\alpha = 0$ represents the upper-bound $u_{i,j}$ and lower-bound $l_{i,j}$ of triangular fuzzy numbers, and $\alpha = 1$ represents the geometric mean $M_{i,j}$ in triangular fuzzy numbers. Thus, b can be viewed as the degree of pessimism in a decision maker. When $\beta = 0$, the decision maker is more optimistic, and the expert consensus is thus upper-bound $u_{i,j}$ of the triangular fuzzy number. When $\beta = 1$, the decision maker is pessimistic, and the number ranges from 0 to 1.
The single pair-wise comparison matrix is expressed in formula

$$g_{\alpha,\beta}(\tilde{A}) = g_{\alpha,\beta}([\tilde{a}_{ij}]) = \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} \begin{bmatrix} 1 & g_{\alpha,\beta}(\tilde{a}_{12}) & \cdots & g_{\alpha,\beta}(\tilde{a}_{1n}) \\ 1/g_{\alpha,\beta}(\tilde{a}_{12}) & 1 & \cdots & g_{\alpha,\beta}(\tilde{a}_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ 1/g_{\alpha,\beta}(\tilde{a}_{1n}) & 1/g_{\alpha,\beta}(\tilde{a}_{2n}) & \cdots & 1 \end{bmatrix}.$$

3. *Determine eigenvector*
$\lambda_{max}$is defined as the eigenvalue of the single pair-wise comparison matrix $g_{\alpha,\beta}(\tilde{A})$

$$g_{\alpha,\beta}(\tilde{A}).W = \lambda_{max}W \quad (4)$$

$$[g_{\alpha,\beta}(\tilde{A}) - \lambda_{max}I]W = 0 (5)$$

Where W denotes the eigenvector of$g_{\alpha,\beta}(\tilde{A})$
The traditional AHP only uses a specific figure (geometric mean) to represent expert opinions in the pair-wise comparison matrix. However, triangular fuzzy numbers represent fuzzy opinions and expert consensus. Meanwhile, both approaches use the eigenvector method to calculate weights.

C. *Consistency test*
A consistency index (C.I.) and consistency ratio (C.R.) to
verifythe consistency of the comparison matrix. The C.I. and R.I. aredefined as follows:

$$C.I= \frac{\lambda_{max}-1}{n-1}, \quad (6)$$

$$C.R=\frac{C.I}{R.I} \quad (7)$$

Where R.I. represents the average consistency index over numerous random entries of the same order reciprocal matrices. If C.R ≤ 0.1, the estimate is accepted; otherwise, a new comparison matrix is solicited until C.R≤ 0.1.

IV. A Case study

The objective of our case study is to select car with three alternative i.e. style, reliability and fuel economy. Each alt-native have three sub-alternatives model A, B, C, D. Using Fuzzy AHP approach we select best car among all. Firstly we collect the information and then arranged in hierarchical tree(see fig.2). Secondly ranking all criteria with triangular fuzzy number and form pair-wise comparison matrix.Using formulae 2 &3 defuzzificationof matrix. Determine eigenvector and verify the consistency ratio.
The table3 show the ranking of alternative using the fuzzy triangular number.
The table 4 indicate weight vector for alternative after using formulae 2, 3& 5
After comparing all criteria with one another and solving matrix calculation we determine weight vector for all criteria
D model is the highest ranking car model. The table5 indicate
Summarizes the resultsweightvector
The consistency of each comparison matrix is tested by formulae (6) and (7). The consistency test results, the C.R. of the compare- sion matrix from each of the eleven experts, are all smaller than ‘‘0.1’’.

Table3 ranking of alternative with TFN

	style	reliability	Fuel economy
style	(1,1,1)	(1/3,1/2,1/1)	(2,3,4)
reliability	(1,2,3)	(1,1,1)	(3.4.5)
Fuel economy	(1/4,1/3,1/2)	(1/5,1/4,1/3)	(1,1,1)

Table4 weight vector of alternative

	style	reliability	Fuel economy	Weight vector
style	1	.5825	3.0000	.4230
reliability	2.0000	1	4.0000	.4779
Fuel economy	.3100	.2500	1	.1030

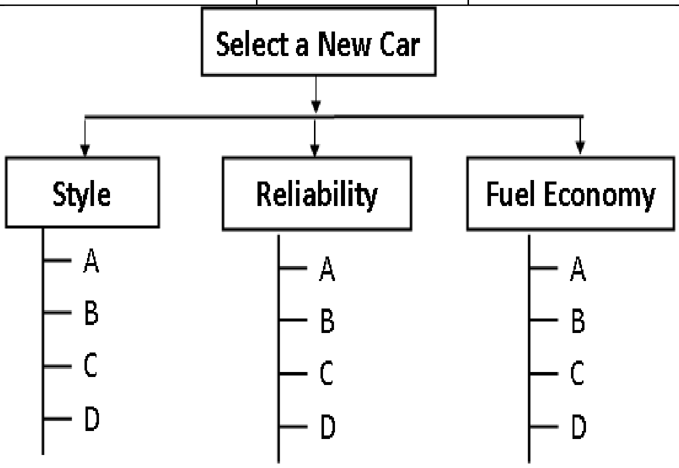

Fig. 2 Hierarchical tree

V.Conclusion

A fuzzy AHP approach that can help us to reach an effective decision using triangular fuzzy number. The objective of this paper was to select the best car model from market .Paper based on fuzzy AHP model, we presented a framework for decision making about best car model .The result of his study show that factor related to style, reliability and fuel economy are the major criteria for assessment and choosing the best car.

Table 5 weight vector criteria base on fuzzy AHP

Criteria	Weight vector	Sub alternative	Weight vector
style	.4230	A	0.1070
		B	.2601
		C	.0480
		D	.5847
reliability	.4779	A	.3590
		B	.2861
		C	.0930
		D	.2610
Fuel economy	.1030	A	.3010
		B	.2390
		C	.2120
		D	.2480

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